Exercise 7.20

Introduction to Discrete Mathematics
MATH 2001

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 7.20 from Hammack [Ham13, Ch. 7]:

Exercise 7.20. Prove the following statement: *There exists an* $n \in \mathbb{N}$ *for which* $11 \mid 2^n - 1$.

Solution. If we consider 2^n modulo 11 (i.e., we consider the remainder of 2^n when divided by 11), we obtain the following table:

n	1	2	3	4	5	6	7	8	9	10
$2^n \pmod{11}$	2	4	8	5	10	9	7	3	6	1

Therefore, it follows that $2^{10} - 1 \equiv 0 \pmod{11}$; i.e., 11 divides $2^{10} - 1$. □

Remark 0.1. One can actually deduce from this proof the stronger statement that given $n \in \mathbb{N}$, one has $11 \mid 2^n - 1$ if and only if $n \equiv 10 \pmod{11}$.

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REFERENCES

[Ham13] Richard Hammack, *Book of proof*, Creative Commons, 2013.

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