

Exercise 3.9.4

Introduction to Discrete Mathematics MATH 2001

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ABSTRACT. This is Exercise 3.9.4 from Hammack [[Ham13](#), §3.9]:

Exercise 3.9.4. Select any five points inside a square whose side-length is one unit. Show that at least two of these points are within $\sqrt{2}/2$ units of each other.

Solution. Divide the square into four smaller squares of side length $1/2$ (i.e., use the perpendicular bisectors of each side of the square to cut it into 4 smaller squares with side length $1/2$). By the division principle, if one puts 5 points into the square, at least two must lie in one of the four smaller squares. The maximum distance between two points in a square is the distance between two opposite corners; in a square of side length $1/2$, this distance is $\sqrt{2}/2$. \square

REFERENCES

[Ham13] Richard Hammack, *Book of proof*, Creative Commons, 2013.

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