Exercise 3.8.11

Introduction to Discrete Mathematics
MATH 2001

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ABSTRACT. This is Exercise 3.8.11 from Hammack [Ham13, §3.8]:

Exercise 3.8.11. How many integer solutions does the equation $w+x+y+z=100$ have if $w \geq 4$, $x \geq 2$, $y \geq 0$, and $z \geq 0$?

Solution. If we consider the equation $w+x+y+z = 100$, and look for non-negative integer solutions, then we see that we are looking for 4 non-negative integers that sum to 100. We can easily solve this using stars and bars, by placing 100 stars and 3 bars (the number of stars in the first range is w, the number of stars in the second range is x, etc.).

However, we want solutions with $w \geq 4$ and $x \geq 2$ (and $y \geq 0$ and $z \geq 0$). We can solve this using stars and bars, as well, by placing $100-4-2 = 94$ stars and 3 bars. In this case, w is four more than the number of stars in the first range, x is two more than the number of stars in the second range, y is the number of stars in the third range, and z is the number of stars in the fourth range. Therefore, there are

$$\binom{94+3}{3} = \binom{97}{3}$$

solutions to the equation $w+x+y+z=100$ with $w \geq 4$, $x \geq 2$, $y \geq 0$, and $z \geq 0$. □

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References

[Ham13] Richard Hammack, *Book of proof*, Creative Commons, 2013.

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