Exercise 11.5.6

Introduction to Discrete Mathematics
MATH 2001

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ABSTRACT. This is Exercise 11.5.6 from Hammack [Ham13, §11.5]:

Exercise 11.5.6. Suppose $[a], [b] \in \mathbb{Z}_6$ and $[a] \cdot [b] = [0]$. Can we conclude that $[a] = [0]$ or $[b] = [0]$? What if $[a], [b] \in \mathbb{Z}_7$?

Solution. Suppose $[a], [b] \in \mathbb{Z}_6$ and $[a] \cdot [b] = [0]$. We cannot conclude that $[a] = [0]$ or $[b] = [0]$, since we have the example $[2] \cdot [3] = [6] = [0]$, but $[2] \neq [0]$ and $[3] \neq [0]$.

On the other hand, suppose $[a], [b] \in \mathbb{Z}_7$ and $[a] \cdot [b] = [0]$. Then we *can* conclude that $[a] = [0]$ or $[b] = [0]$. Indeed, if $[a] \cdot [b] = [0]$, then $ab = 7n$ for some integer n. Since 7 is *prime*, from unique factorization of integers, we can conclude that 7 divides a or 7 divides b; i.e., $[a] = [0]$ or $[b] = [0]$. $\square$

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REFERENCES

[Ham13] Richard Hammack, *Book of proof*, Creative Commons, 2013.

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