Exercise 11.3.4

Introduction to Discrete Mathematics
MATH 2001

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 11.3.4 from Hammack [Ham13, §11.3]:

Exercise 11.3.4. Let $A = \{a, b, c, d, e\}$. Suppose R is an equivalence relation on A. Suppose also that $a \sim d$ and $b \sim c$, $e \sim a$ and $c \sim e$. How many equivalence classes does R have?

Solution. There is one equivalence class. In particular, $A = \{a, b, c, d, e\} = [a]$. Indeed, by definition,

$$[a] = \{x \in A : x \sim a\}.$$

We clearly have $[a] \subseteq A$, so we just need to show the opposite containment. First, we have $a \in [a]$, since $a \sim a$ (reflexive property of an equivalence relation). Next, we have $b \in [a]$. To see this, observe that we are given $b \sim c$ and $c \sim e$, which implies that $b \sim e$ (transitive property of an equivalence relation). Using this with the given equivalence $e \sim a$, we have $b \sim a$ (again by the transitive property of an equivalence relation). Next we have $c \in [a]$, since we are given $c \sim e$ and $e \sim a$ (where again we are concluding by the transitive property of an equivalence relation). We also have $d \in [a]$, since we are given $a \sim d$, and this implies that $d \sim a$ (symmetric property of an equivalence relation). Finally, we are given $e \sim a$, so that $e \in [a]$. □

Date: November 23, 2024.

REFERENCES

[Ham13] Richard Hammack, *Book of proof*, Creative Commons, 2013.

UNIVERSITY OF COLORADO, DEPARTMENT OF MATHEMATICS, CAMPUS BOX 395, BOULDER, CO 80309

Email address: casa@math.colorado.edu