Exercise 11.2.2

Introduction to Discrete Mathematics
MATH 2001

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ABSTRACT. This is Exercise 11.2.2 from Hammack [Ham13, §11.2]:

Exercise 11.2.2. Consider the relation $R = \{(a,b),(a,c),(c,c),(b,b),(c,b),(b,c)\}$ on the set $A = \{a,b,c\}$. Is R reflexive? Symmetric? Transitive? If a property does not hold, say why.

Solution. The relation R is not reflexive, as $(a,a) \notin R$. The relation is not symmetric, as $(a,b) \in R$ but $(b,a) \notin R$.

The relation R is transitive. We need to consider every pair (x,y) and (y,z) in R and check that (x,z) is in R.

We start with (a,b). Then we have (b,b) and (b,c) to check. In other words, we have (a,b) and (b,b) in R, and so we must check that (a,b) is in R, which is the case. Similarly, we have (a,b) and (b,c) in R, so we must check that (a,c) is in R, which is the case.

Next we look at (a,c). Then we have to check (c,c) and (c,b). We see that this is OK, since (a,c) and (a,b) are in R.

Next we look at (c,c). Then we have to check (c,c) and (c,b). We see this is OK as (c,c) and (c,b) are in R.

Next we look at (b,b). Then we have to check (b,b) and (b,c). We see this is OK as (b,b) and (b,c) are in R.

Next we look at (c,b). Then we have to check (b,b) and (b,c). We see that this is OK as (c,b) and (c,c) are in R.

Last we look at (b,c). Then we have to check (c,c) and (c,b). We see that this is OK as (b,c) and (b,b) are in R. □

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References

[Ham13] Richard Hammack, *Book of proof*, Creative Commons, 2013.

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