Final Exam

Intro to Discrete Math

MATH 2001

Spring 2022

Tuesday May 3, 2022

NAME: ______________________________

PRACTICE EXAM

Question:	**1**	**2**	**3**	**4**	**5**	Total
Points:	20	20	20	20	20	100
Score:						

- The exam is closed book. You **may not use any resources** whatsoever, other than paper, pencil, and pen, to complete this exam.
- You **may not discuss the exam** with anyone except me, in any way, under any circumstances.
- You **must explain your answers**, and you will be **graded on the clarity of your solutions**.
- You must upload your exam as a single **.pdf** to **Canvas**, with the questions in the correct order, etc.
- You have 60 minutes to complete the exam.

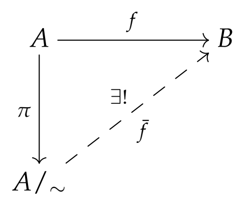
1. (20 points) • Let A be a set, and let $\sim$ be an equivalence relation on A.

Recall that for each $a \in A$, we denote by $[a]$ the equivalence class of A, and by $A/_\sim$ the set of equivalence classes of A. There is a map of sets $\pi : A \to A/_\sim$ defined by $a \mapsto [a]$.

Show that $\pi : A \to A/_\sim$ **satisfies the following property:**

> *If* $f : A \to B$ *is a map of sets such that for all* $a_1, a_2 \in A$ *with* $a_1 \sim a_2$ *we have* $f(a_1) = f(a_2)$*, then there exists a unique map of sets* $\bar{f} : A/_\sim \to B$ *such that* $\bar{f} \circ \pi = f$.

The following diagram may be helpful in thinking about this:



[*Hint*: show that the rule $\bar{f}([a]) = f(a)$ for all $[a] \in A/_\sim$ defines a map $\bar{f}$ with the desired properties.]

1
20 points

2. (20 points) • **TRUE** or **FALSE**:

Let S be a set, and let $\mathcal{A} \subseteq \mathscr{P}(S)$ be a set of subsets of S. Define a relation on $\mathcal{A}$ by the rule that for $A, B \in \mathcal{A}$, we have $A \leq B$ if $|A| \leq |B|$.

The relation $\leq$ gives $\mathcal{A}$ the structure of a POSET; i.e., $(\mathcal{A}, \leq)$ is a POSET.

If the statement in italics is true, give a proof. If the statement is false, provide a counter example, and prove that it is a counter example.

Your solution must start with the sentence, "*This statement is TRUE*," or the sentence, "*This statement is FALSE*."

2
20 points

3. (20 points) • **TRUE** or **FALSE**:

For sets A and B, if there is an injection $f : A \to B$ and a surjection $g : A \to B$, then there is a bijection $h : A \to B$.

If true, give a proof. If false, provide a counter example, and prove that it is a counter example.

Your solution must start with the sentence, "*This statement is TRUE*," or the sentence, "*This statement is FALSE*."

3
20 points

4. • A committee of 50 senators is chosen at random (from the full senate consisting of 100 senators). I want to be able to tell people what the probability is that any given collection of senators will be included in the committee.

(a) (10 points) **What probability space should I use to start to answer these types of questions?**

(b) (5 points) **If I give you a set of senators $C \subseteq \{1, 2, 3, \ldots, 100\}$, explain how you can use your probability space to determine the probability that the senators in C are included in the committee.** (For instance, you should be able to tell me the probability that both senators from Colorado are included in the committee.)

(c) (5 points) **What is the probability that both senators from Colorado are included in the committee given that at least one is?**

4
20 points

5. • **TRUE** or **FALSE**. For this problem, and this problem only, **you do <u>not</u> need to justify your answer**.

(a) (4 points) **TRUE** or **FALSE** (circle one). The LaTeX code:

```
There exists $\bar f:A/_\sim \to B$ such that $\bar f([a])=f(a)$.
```

produces the following:

There exists $\bar f:A/_\sim \to B$ such that $\bar f([a])=f(a)$.

(b) (4 points) **TRUE** or **FALSE** (circle one). If $f : A \to B$ is a map of sets and $C \subseteq B$, then we have that $f(f^{-1}(C)) = C$.

(c) (4 points) **TRUE** or **FALSE** (circle one). Given a POSET $(P, \leq)$, a sub-POSET $(P', \leq')$ is called a chain if it has a maximal element.

(d) (4 points) **TRUE** or **FALSE** (circle one). The set $\mathrm{Map}(\mathbb{Q}, \mathbb{Q})$ of maps from $\mathbb{Q}$ to $\mathbb{Q}$ is countable.

(e) (4 points) **TRUE** or **FALSE** (circle one). If $(S, \mathcal{B}, P)$ is a probability space, and $A, B, C \in \mathcal{B}$ are events, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

5
20 points