Exercise 31.1

Abstract Algebra 1
MATH 3140

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 31.1 from Fraleigh [Fra03, §31]:

Exercise 31.1. Find the degree and a basis for the field extension $\mathbb{Q}(\sqrt{2})$ over $\mathbb{Q}$.

Solution. By Eisenstein's Criterion applied to the prime $p = 2$ (or using the fact that $\sqrt{2}$ is not rational), we see that $x^2 - 2 \in \mathbb{Q}[x]$ is irreducible, so that the extension $\mathbb{Q}(\sqrt{2})$ over $\mathbb{Q}$ has degree 2, with basis given by $1, \sqrt{2}$ (see [Fra03, Theorem 29.18] or [Fra03, Theorem 30.23]). □

Date: December 9, 2021.

REFERENCES

[Fra03] John Fraleigh, *A First Course in Abstract Algebra*, Seventh edition, Addison Wesley, Pearson, 2003.

UNIVERSITY OF COLORADO, DEPARTMENT OF MATHEMATICS, CAMPUS BOX 395, BOULDER, CO 80309

Email address: casa@math.colorado.edu