Solutions:

§6.6 Part II: Pressure

(Created by Faan Tone Liu)

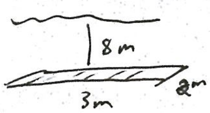
Key Points:

- Pressure $= P = \rho \cdot g \cdot d$
  - $\rho =$ mass density of fluid (pronounced "rho")
  - $g = 9.8 \frac{\text{m}}{\text{sec}^2}$
  - $d =$ depth
  - Units: $\frac{kg}{m^3} \cdot \frac{m}{sec^2} \cdot m = \frac{kg\,m}{sec^2} \cdot \frac{1}{m^2} = N \cdot \frac{1}{m^2} = \frac{N}{m^2} = P_a$ (Pascal)
- Force = Pressure × Area

Examples:

1. A 3m × 2m piece of glass sits horizontally 8m under water. What is the force on each side of the glass?

Constant pressure/Force everywhere, so no calculus!

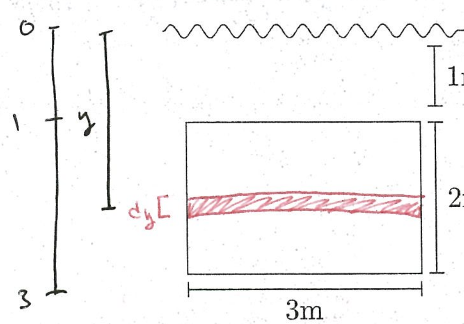


$$F = \overset{\text{pressure}}{P} \cdot \overset{\text{area}}{A}$$
$$= \rho \cdot g \cdot d \cdot 6m^2$$
$$= \frac{1000\,kg}{m^3} \cdot 9.8 \frac{m}{sec^2} \cdot 8m \cdot 6m^2$$
$$= 470400 \text{ N}$$

2. A 3m × 2m piece of glass sits underwater as shown with its top 1m from the surface of the water. Find the force on each side of the glass.

Pressure changes w/ depth!

Strategy: chop into pieces (slices) of width dy and then integrate

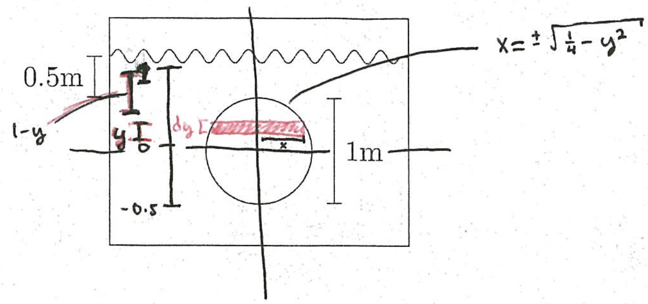


$$P_{slice} = \rho \cdot g \cdot d$$
$$= 1000 \frac{kg}{m^3} \cdot 9.8 \frac{m}{sec^2} \cdot y$$

$$F_{slice} = P_{slice} \cdot A_{slice}$$
$$= 9800y \cdot \widetilde{3 \cdot dy}$$

$$F_{Total} = \int_1^3 3 \cdot 9800y\,dy = \cdots = 117600 \text{ N}$$

3. A round observation window behind the Millenium Hotel looks through a cement wall into boulder creek. What is the force on the window?



$$P_{slice} = \rho \cdot g \cdot d$$

depth of slice

$$= 1000\,\frac{kg}{m^3} \cdot 9.8\,\frac{m}{sec^2} \cdot (1-y)$$

$$= 9800(1-y)$$

$$F_{slice} = P_{slice} \cdot A_{slice}$$

$$= 9800(1-y) \cdot 2x\,dy$$

$$= 9800(1-y) \cdot 2\sqrt{\tfrac{1}{4} - y^2}\,dy$$

$2x$, where

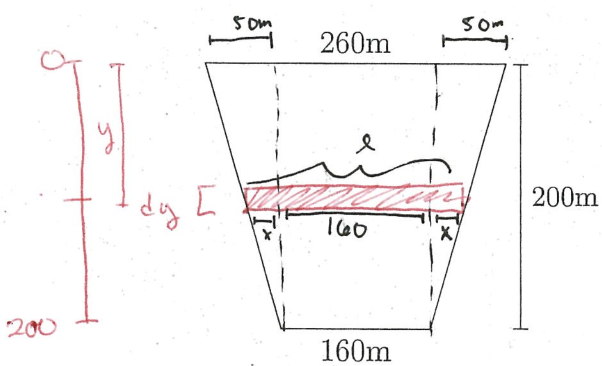
$$x = \sqrt{\tfrac{1}{4} - y^2}$$

$$A_{slice} = 2\sqrt{\tfrac{1}{4} - y^2}$$

$$F_{Total} = \int_{-0.5}^{0.5} 9800(1-y)\, 2\sqrt{\tfrac{1}{4} - y^2}\, dy$$

$= \cdots$ in Newtons

4. The approximate dimensions of Hoover Dam are shown. Model it as a trapezoid, and calculate the force the water pressure puts on the dam.



$$\frac{x}{50} = \frac{200-y}{200}$$

$$x = \tfrac{1}{4}(200-y)$$

(could also find an equation for the line and then do similar to #3)

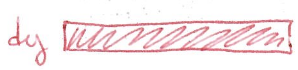
$$P_{slice} = \rho \cdot g \cdot d$$

$$= 1000 \cdot 9.8 \cdot y$$

$$= 9800y$$

$$F_{slice} = P_{slice} \cdot A_{slice}$$

$$= 9800y \cdot (160 + \tfrac{1}{2}(200-y))\,d$$



$$l = 160 + 2x$$

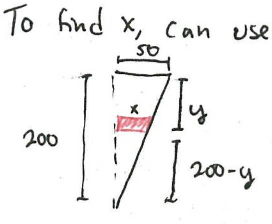
$$F_{Total} = \int_0^{200} 9800y\,(160 + \tfrac{1}{2}(200-y))\,d$$

$$A_{slice} = l \cdot dy$$

$$= (160 + 2 \cdot \tfrac{1}{4}(200-y))\,dy$$

$= \cdots$ in Newtons.

To find x, can use similar triangles:



⚠ There are many correct ways to do this [e.g. could make "y" what we called "200-y"] ...