Polar Coordinates (Appendix H1)

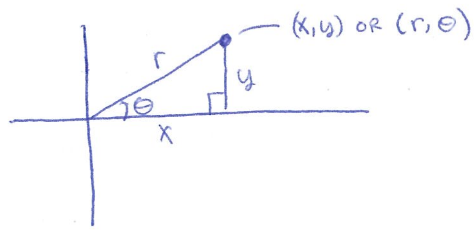
Thanks to Faan Tone Liu

Key Points:

- Every location in the plane can be described by (r, θ), where

$$r = \text{distance from the origin}$$
$$\theta = \text{angle from the positive } x\text{-axis.}$$

- Consider the following picture:



- Converting from polar to rectangular coordinates:

$$x = r\cos\theta$$
$$y = r\sin\theta$$

- Converting from rectangular to polar coordinates:

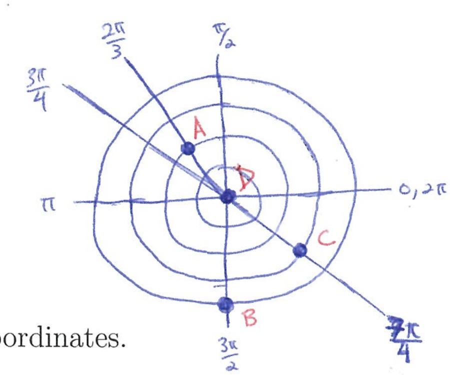
$$r^2 = x^2+y^2$$
$$\tan\theta = y/x$$

(Also have $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$)

Examples:

1. Plot the following points:

   A. $(r, \theta) = (2, \frac{2\pi}{3})$
   B. $(r, \theta) = (4, \frac{3\pi}{2})$
   C. $(r, \theta) = (-3, \frac{3\pi}{4})$
   D. $(r, \theta) = (0, \frac{11\pi}{6})$

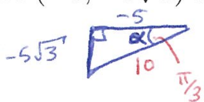


2. Convert $(2, \frac{2\pi}{3})$ into rectangular coordinates.

$$x = r\cos\theta = 2\cdot\cos\left(\frac{2\pi}{3}\right) = 2\cdot\frac{-1}{2} = -1$$
$$y = r\sin\theta = 2\sin\left(\frac{2\pi}{3}\right) = 2\cdot\frac{\sqrt{3}}{2} = \sqrt{3}$$

$\boxed{(-1, \sqrt{3})}$ (see A above)

3. Convert $(-5, -5\sqrt{3})$ into polar coords.



(Not to scale)

$r^2 = x^2 + y^2 = 25 + 25 \cdot 3 = 100$

$r = 10$

$\tan\alpha = \frac{-5\sqrt{3}}{-5} = \sqrt{3}$

$\alpha = \frac{\pi}{3}$

Angle in Quad III, so $\theta = \frac{4\pi}{3}$. $\Rightarrow$ $\boxed{\left(10, \frac{4\pi}{3}\right)}$

4. Convert $r = 2$ to rectangular coords.

$x^2 + y^2 = r^2$

$\boxed{x^2 + y^2 = 4}$ ← Circle of radius 2

5. Convert $r = 3\cos\theta$ to rectangular coords.

$\left[\begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \\ x^2 + y^2 = r^2 \end{array}\right]$

↑ Dictionary

$\frac{x}{r} = \cos\theta$, so we have

$r = 3 \cdot \frac{x}{r}$

$r^2 = 3x$

$x^2 + y^2 = r^2$, so, now,

$x^2 + y^2 = 3x$ $\Rightarrow$ $x^2 - 3x + y^2 = 0$

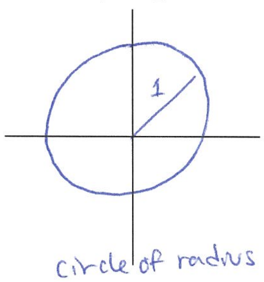
$x^2 - 3x + \frac{9}{4} + y^2 = \frac{9}{4}$

$\boxed{\left(x - \frac{3}{2}\right)^2 + y^2 = \frac{9}{4}}$

circle of radius $\frac{3}{2}$ centered at $\left(\frac{3}{2}, 0\right)$

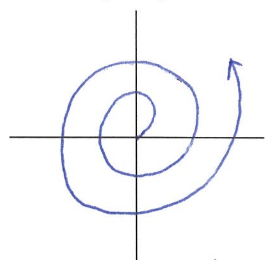
6. Graph the following polar curves:

$r = 1$



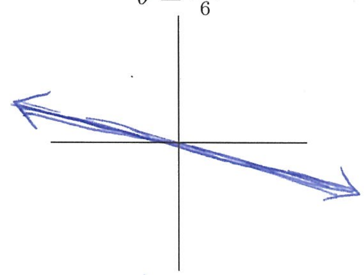
circle of radius 1

$r = \theta$



a spiral

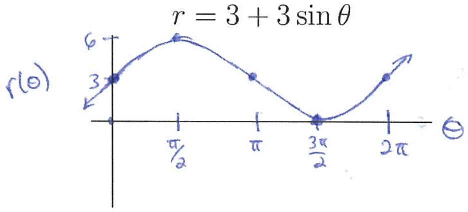
$\theta = \frac{5\pi}{6}$



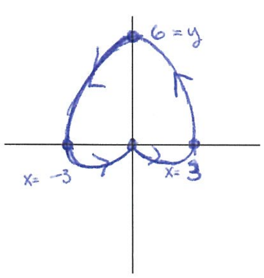
a line

7. Graph the following polar curves (Hint: first graph in rect. coords):

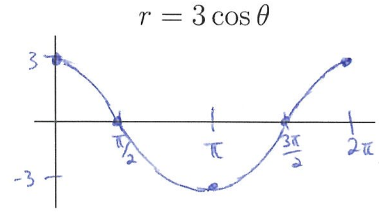
$r = 3 + 3\sin\theta$

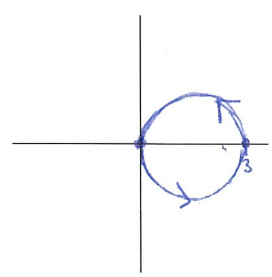


This tells us about the radius.



$r = 3\cos\theta$





Loops twice between $\theta = 0$ and $\theta = 2\pi$.