§5.7: Partial Fractions Decomposition

(Thanks to Faan Tone Liu)

Key Points:

- Use this method to integrate rational functions.
- Degree of numerator must be lower than degree of denominator. If needed, start with long division.
- Factor the denominator
- Solve the decomposition according to the right form:
  - Linear Factors: $$\frac{5x+3}{(x+1)(x+4)} = \frac{A}{x+1} + \frac{B}{x+4}$$
  - Repeated Linear Factors: $$\frac{2x-4}{(x-2)(x+3)^2} = \frac{A}{x-2} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$$
  - Irreducible Quadratic Factors: $$\frac{2x^2-3x-1}{(x-1)(x^2+9)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+9}$$
  - Mixtures are possible!
- Make sure you remember how to integrate the "outputs" of the partial fractions decomposition. For example:

$$\frac{5}{2x+1}, \ \frac{3}{(x+4)^2}, \ \frac{2x}{x^2+9}, \ \frac{4}{x^2+25}, \ \text{or} \ \frac{3x+1}{x^2+16}.$$

- Other notes and tips:

Examples:

1. $\displaystyle\int \frac{2x^2+3x-3}{x^2-x}\,dx$

   First, use long division to re-write the integrand so the fraction involved has a numerator with a lower degree than the denominator.

   The form of the decomposition of $\frac{5x-3}{x(x-1)}$ is:

   Here are two methods to find A and B in the partial fractions decomposition:

Method I: Equate Coefficients	Method II: Judicious Substitution

   Now, $\displaystyle\int \frac{2x^2+3x-3}{x^2-x}\,dx =$

2. $\displaystyle\int \frac{x^2+x-5}{(x-2)(x-1)^2}\,dx$

3. $\displaystyle\int \frac{10}{(x+1)(x^2+9)}\,dx$