

# Differential Equations: Applications (Ch 7)

## Key Points:

- If the rate at which  $y(t)$  changes is proportional to the value of  $y$  at a time  $t$ , we have the differential equation

$$\frac{dy}{dt} = ky.$$

Some examples where this occurs include:

- **Newton's Law of Cooling** states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings (provided that this difference is not too large). This is summarized in the differential equation

$$\frac{dT}{dt} = k(T - T_s),$$

where  $T = T(t)$  is the temperature of the object at time  $t$ , and  $T_s$  is the temperature of the surroundings.

## Exercises:

1. Strontium-90 has a half-life of 28 days.
  - (a) Write down a differential equation to model this situation. Let  $m = m(t)$  be the mass of strontium after  $t$  days.
  - (b) A sample has a mass of 50 mg initially. Find a formula for the mass remaining after  $t$  days.
  - (c) Find the mass remaining after 40 days.
  - (d) How long does it take the sample to decay to a mass of 2mg?

2. When *E. coli* grows in a nutrient-broth medium each cell divides into two cells every 20 minutes. Suppose the initial population of a culture of *E. coli* is 60 cells.

(a) Find the relative growth rate (i.e.  $k$ ).

(b) Find an expression for the number of cells after  $t$  hours.

(c) Find the number of cells after 8 hours.

(d) Find the rate of growth after 8 hours.

(e) When will the population reach 20,000 cells?

3. A freshly brewed cup of coffee has temperature  $95^\circ\text{C}$  in a  $20^\circ\text{C}$  room. When its temperature is  $70^\circ\text{C}$ , it is cooling at a rate of  $1^\circ\text{C}$  per minute. When does this occur?

4. Experiments show that the reaction  $\text{H}_2 + \text{Br}_2 \rightarrow 2\text{HBr}$  satisfies the rate law

$$\frac{d[\text{HBr}]}{dt} = k[\text{H}_2][\text{Br}_2]^{1/2},$$

where  $[\text{HBr}]$ ,  $[\text{H}_2]$ , and  $[\text{Br}_2]$  represent the concentrations of molecules of Hydrogen Bromide, Hydrogen, and Bromine, respectively. Suppose that the initial concentrations of the reactants are  $[\text{H}_2]_0 = a$  moles/L and  $[\text{Br}_2]_0 = b$  moles/L.

- (a) Write a differential equation that relates the rate of change of the concentration  $[\text{HBr}]$  to the concentrations  $[\text{H}_2]$  and  $[\text{Br}_2]$ . (**Hint:** Let  $x = x(t)$  be the concentration of  $[\text{HBr}]$  at time  $t$ .)
- (b) Find  $x$  as a function of  $t$  in the case where  $a = b$ . Use the fact that  $x(0) = 0$  (Why does this make sense?).