§7.5: The Logistic Equation

(Thanks to Faan Tone Liu)

Key Points:

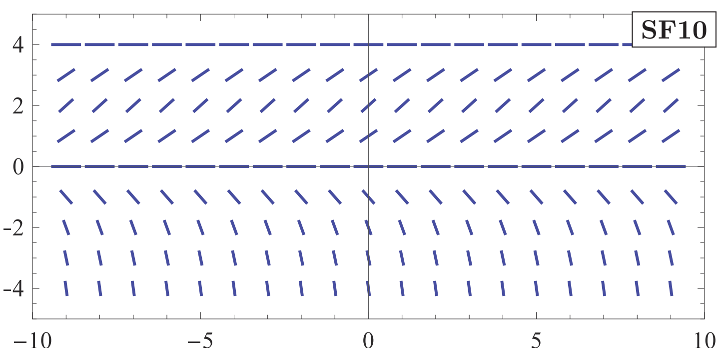
- Review: Often, population growth can be modeled by $P' = kP$. The solution is __________________, and this situation is called __________________________.

- Exponential growth is not realistic in the long run because $\lim_{t\to\infty} P(t) =$ _______, so we modify it to get the **Logistic Equation**

$$\frac{dP}{dt} = \qquad\qquad\qquad\qquad ,$$

  where M is a constant that represents the carrying capacity of the population. The solution is

- The slope field for the logistic growth equation is



- Miscellaneous observations:
  - If P is small, then $\frac{dP}{dt} \approx$ ______ (basically exponential growth).
  - If $P \approx M$, then $\frac{dP}{dt} \approx$ ______ (growth slows to 0).
  - Equilibrium (constant) solutions are: _________________________.
  - If the population starts between 0 and M: $\lim_{t\to\infty} P(t) =$ ______.
  - Using Calc I methods, we can show that $P(t)$ has an inflection point when ______.

Examples:

1. (Review of exponential growth) Solve the differential equation $P' = kP$.

2. Solve the logistic differential equation $P' = kP\left(1 - \frac{P}{M}\right)$.

3. Suppose a population grows according to the logistic model with an initial population of 1000 and a carrying capacity of 10,000. If the population grows to 2500 after one year, what is the population after three more years?