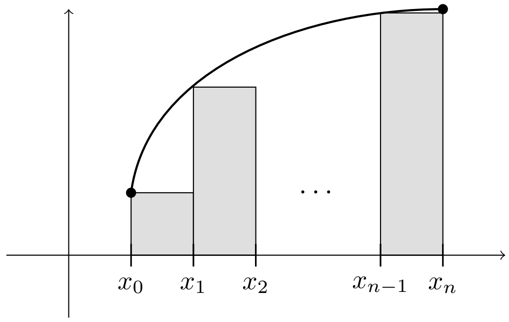
Background info, approximating $\int_a^b f(x)\,dx$.

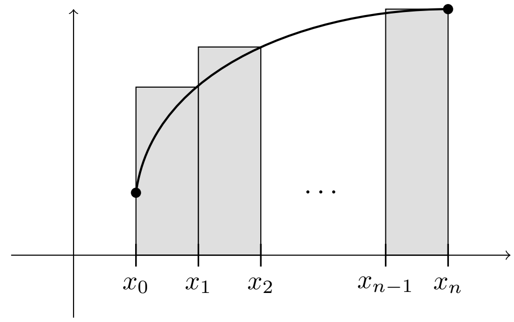
For each method, the subintervals are uniform. That is, $a = x_0$, $b = x_n$, and $\Delta x = \frac{b-a}{n}$.

Left-endpoint approximation



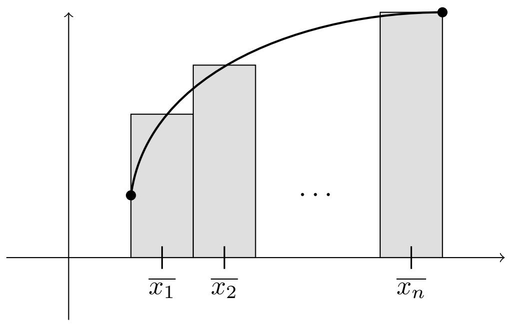
$$L_n = \Delta x[f(x_0) + f(x_1) + \cdots + f(x_{n-1})]$$

Right-endpoint approximation



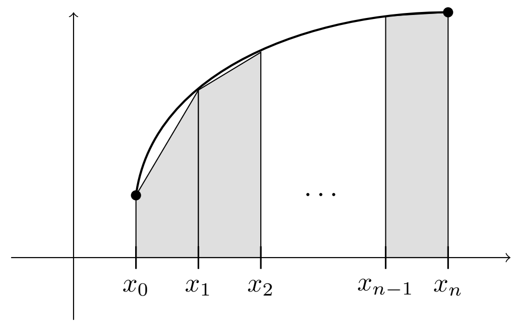
$$R_n = \Delta x[f(x_1) + f(x_2) + \cdots + f(x_n)]$$

Midpoint approximation



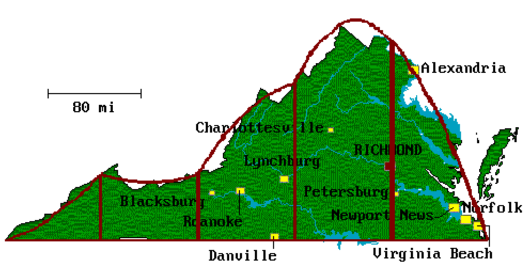
$$M_n = \Delta x[f(\overline{x_1}) + f(\overline{x_2}) + \cdots + f(\overline{x_n})]$$

Trapezoidal approximation



$$T_n = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + f(x_n)]$$
$$T_n = \frac{1}{2}(L_n + R_n)$$

Simpson's rule (note, n must be even). Simpson's rule uses sections of parabolas to estimate areas. For more about this image see `http://www.maa.org/publications/periodicals/loci/joma/estimating-the-area-of-virginia-using-simpsons-rule`



$$S_n = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$
$$S_{2n} = \frac{1}{3}(T_n + 2M_n)$$

1. Values of $f(x)$ are given in the table below:

x	5	7	9	11	13	15	17
$f(x)$	-2	0	1	3	4	5	8

Estimate $\int_5^{17} f(x)\,dx$ using the following methods, if possible.

With $n = 3$, $L_n =$

With $n = 6$, $R_n =$

With $n = 6$, $T_n =$

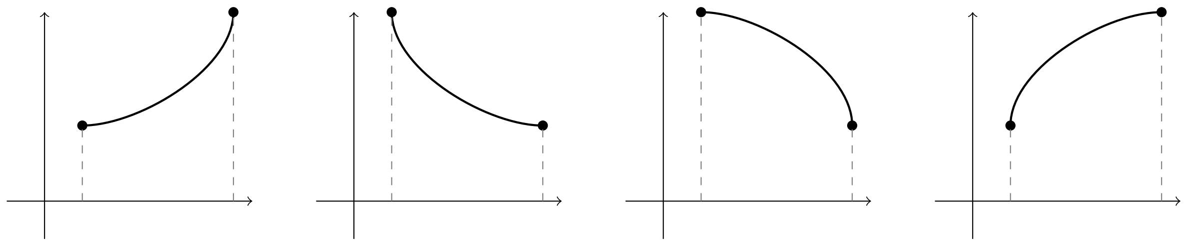
With $n = 6$, $M_n =$

With $n = 3$, $M_n =$

With $n = 3$, $S_n =$

With $n = 6$, $S_n =$

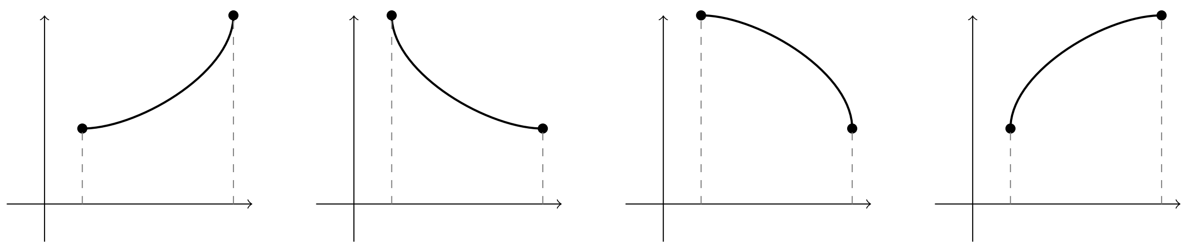
2. a. Examples of L_n. Please draw rectangles for $n = 2$.



When $f(x)$ is ________________________, L_n is an overestimate.

When $f(x)$ is ________________________, L_n is an underestimate.

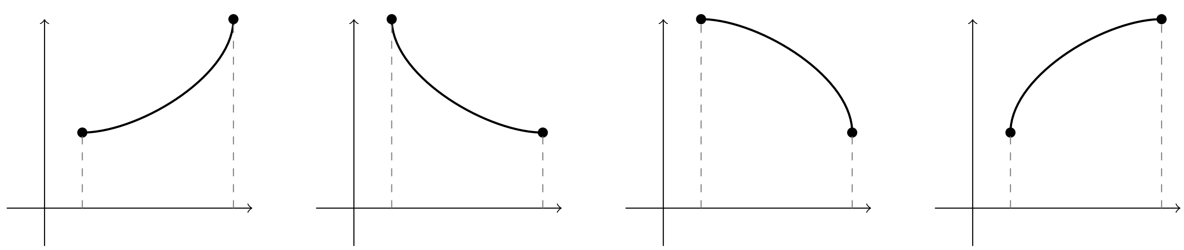
b. Examples of R_n. Please draw rectangles for $n = 2$.



When $f(x)$ is ________________________, R_n is an overestimate.

When $f(x)$ is ________________________, R_n is an underestimate.

c. Examples of T_n. Please draw trapezoids for $n = 2$.

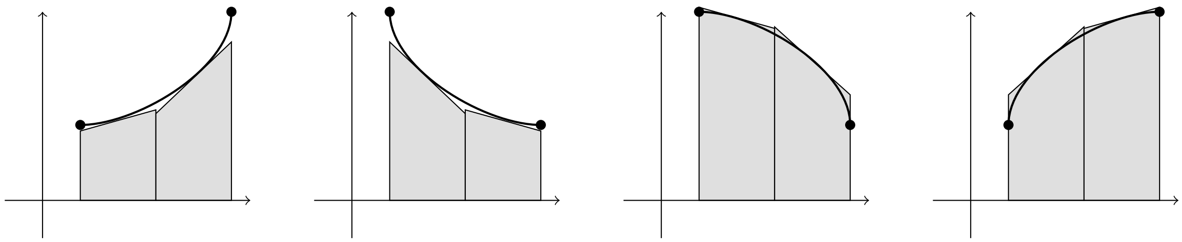


When $f(x)$ is ________________________, T_n is an overestimate.

When $f(x)$ is ____________________________, T_n is an underestimate.

2. d. Examples of M_n, with $n = 2$.

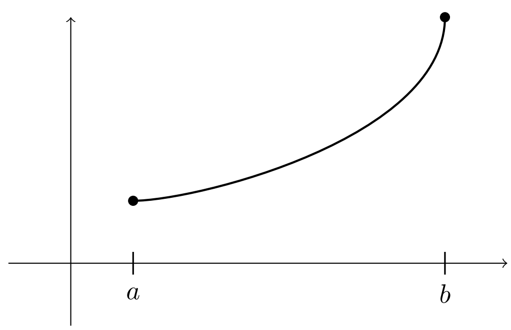
   By 'rotating' the top edge of the rectangles of a Midpoint approximation, we can draw them as trapezoids.



   When $f(x)$ is __________________________, M_n is an overestimate.

   When $f(x)$ is ________________________, M_n is an underestimate.

3. For $f(x)$ shown below, put L_n, R_n, M_n, T_n and $\int_a^b f(x)\,dx$ in order from smallest to largest.



__________ < __________ < _____________ < __________ < __________

Background info, error bounds (see p.405 in the textbook).

> Suppose $|f''(x)| \leq k$ for $a \leq x \leq b$. If E_T and E_M are the errors in the trapezoidal and midpoint approximations, then
>
> $$|E_T| \leq \frac{k(b-a)^3}{12n^2} \quad \text{and} \quad |E_M| \leq \frac{k(b-a)^3}{24n^2}$$

Example 1: If we use the trapezoidal approximation with $n = 10$ to estimate $\int_1^3 x^3\,dx$, how accurate are we guaranteed to be? (If you want, make a guess before you do the calculation.)

$f(x) = x^3$

$f'(x) =$_________

$f''(x) =$________

On $[1, 3]$, $|f''(x)| \leq$________, because ____

So, $|E_T| \leq$ __________________________ (Is this more or less accurate than you guessed?)

Example 2: If we use the midpoint approximation with $n = 20$ to estimate $\int_0^1 \sin(2x)\,dx$, how accurate are we guaranteed to be?

Example 3: How large should n be to guarantee that using T_n to estimate $\int_0^1 e^{-3x}\,dx$ gives an error no larger than 0.001?