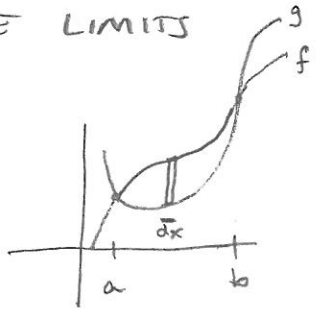
CALC 2 - REVIEW/PREVIEW UNIT 9

FINDING THE AREA BOUNDED BETWEEN CURVES

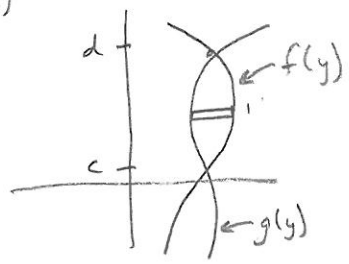
KEY POINTS:

- GRAPH THE FUNCTIONS
- LOCATE THE INTERSECTION POINTS, THESE GIVE LIMITS OF INTEGRATION.
- VERTICAL SLICES: $\text{Area} = \int_a^b \underbrace{f(x) - g(x)}_{\text{top - bottom = height of rectangle}} \underbrace{dx}_{\text{width of rectangle}}$



- HORIZONTAL SLICES:
  SOLVE EQUATIONS FOR x. $(x_1 = g(y),\ x_2 = f(y))$

$$\text{Area} = \int_c^d \underbrace{f(y) - g(y)}_{\text{right - left.}}\, dy$$



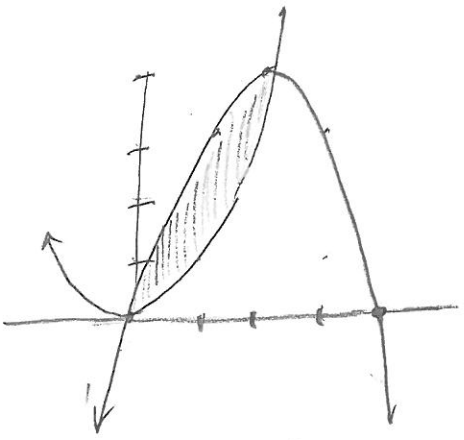
EXAMPLE 1: FIND THE AREA BOUNDED BETWEEN THE CURVES $y = x^2$ AND $y = 4x - x^2$

GRAPH: $y = x^2$ IS A STANDARD PARABOLA.

$y = 4x - x^2 = x(4-x) \Rightarrow$ ZEROES AT $x = 0$ AND $x = 4$, OPENS DOWN.

INTERSECTION:

$$x^2 = 4x - x^2$$
$$2x^2 - 4x = 0$$
$$2x(x-2) = 0$$
$$x = 0;\ x = 2$$



REGION IS BOUNDED IN A 2x4 RECTANGLE. I GUESS AREA IS LESS THAN HALF OF 8.

$$\text{AREA} = \int_0^2 \underbrace{\text{top - bottom}}_{\text{height}} \cdot \underbrace{dx}_{\text{width}}$$

$$= \int_0^2 (4x - x^2) - x^2\, dx$$

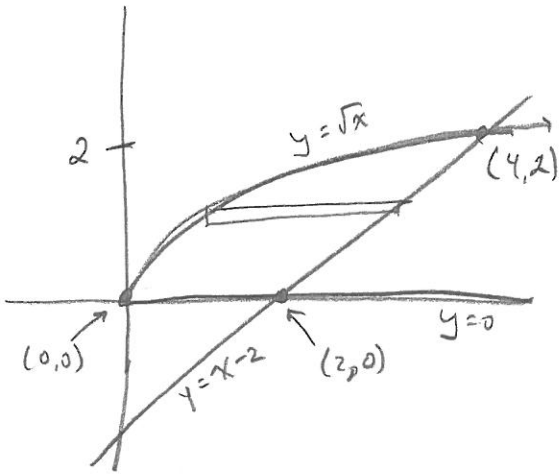
$$= \int_0^2 4x - 2x^2\, dx$$

$$= \left(2x^2 - \tfrac{2}{3}x^3\right)\Big|_0^2 = 8 - \frac{16}{3} - (0-0) = \frac{8}{3} \text{ square units.}$$

(WHICH IS CONSISTENT WITH MY ESTIMATE)

EXAMPLE 2: FIND THE AREA IN THE FIRST QUADRANT BOUNDED BY $y = \sqrt{x}$, $y = 0$, $y = x - 2$

GRAPH EACH FUNCTION. THERE ARE INTERSECTION POINTS AT $(0,0)$ AND $(2,0)$. THIRD INTERSECTION:



$$\sqrt{x} = x - 2$$
$$x = (x-2)^2$$
$$x = x^2 - 4x + 4$$
$$0 = x^2 - 5x + 4$$
$$0 = (x-4)(x-1)$$
$$x = 4, \quad \cancel{x = 1}$$
$$y = 2, \quad \cancel{y = -1}$$

Extraneous solution introduced when we squared $\sqrt{x}$.

WE WILL SLICE HORIZONTALLY.

FIRST SOLVE FOR x:

$$\begin{cases} x = y^2 \leftarrow \text{left} \\ x = y + 2 \leftarrow \text{right} \end{cases}$$

$$\text{Area} = \int_{y=0}^{y=2} (y+2) - y^2 \, dy = \frac{y^2}{2} + 2y - \frac{y^3}{3} \Big|_0^2 = 2 + 4 - \frac{8}{3} = \frac{10}{3}$$

EXERCISES:

1. THE CURVES $y = x^3 - x^2$ AND $y = 2x$ BOUND TWO REGIONS. FIND THE AREA OF THE LARGER.

2. GRAPH THE REGION BOUNDED BETWEEN $x = y^2 - 4y$ AND $x = 2y - y^2$ AND FIND THE AREA.

3. REDO EXAMPLE 2, THE AREA BETWEEN $y = \sqrt{x}$, $y = 0$ AND $y = 2$, BUT SLICING VERTICALLY

4. FIND THE AREA OF THE REGION BOUNDED BY $y = \ln x$, $y = 0$ AND $x = e$. TAKE HORIZONTAL SLICES.