CALCULUS 2 – REVIEW/PREVIEW UNIT 7
COMPLETING THE SQUARE AND INVERSE TRIG INTEGRALS

MOTIVATION: $\int \frac{dx}{(x+3)^2+4}$ $\begin{cases} u = x+3 \\ du = dx \end{cases}$

$= \int \frac{du}{u^2+4}$

$= \frac{1}{2}\arctan\left(\frac{u}{2}\right)+C = \frac{1}{2}\arctan\left(\frac{x+3}{2}\right)+C$

BUT $\int \frac{dx}{\underline{(x+3)^2+4}} = \int \frac{dx}{x^2+6x+9+4} = \int \frac{dx}{x^2+6x+13}$

So Given integrals like $\int \frac{dx}{x^2+6x+13}$
we complete the square
to make it obviously an arctan integral

Precalc review: completing the square

Example 1: $x^2+8x+25$

$= (x^2+\underline{8}x \qquad) + 25$

$= (x^2+8x+\underline{16}) + 25 - \underline{16}$

$= (x^2+8x+16) + 9$

$= (x+4)(x+4) + 9$

$= (x+4)^2+9$

EXAMPLE 2:

$\int \frac{1}{x^2-6x+34}\,dx$

$= \int \frac{1}{(x-3)^2+25}\,dx$ $\begin{cases} u = x-3 \\ du = dx \end{cases}$

$= \int \frac{1}{u^2+25}\,dx$

$= \frac{1}{5}\arctan\left(\frac{x-3}{5}\right)+C$

complete the square:

$x^2-6x+34$

$= (x^2-6x+9)+34-9$

$= (x-3)^2+25$

EXAMPLE 3:

$$\int \frac{1}{\sqrt{15-2x-x^2}}\,dx$$

$$= \int \frac{1}{\sqrt{16-(x+1)^2}}\,dx$$

$$\begin{cases} u = x+1 \\ du = dx \end{cases}$$

$$= \int \frac{1}{\sqrt{16-u^2}}\,du$$

$$= \arcsin\left(\frac{x+1}{4}\right) + C$$

Recall:

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\left(\frac{x}{a}\right)+c$$

complete the square

$$15-2x-x^2$$

$$= 15-(x^2+2x+1)+1$$

$$= 16-(x^2+2+1)$$

$$= 16-(x+1)^2$$

Exercises:

1. $$\int \frac{1}{x^2+4x+29}\,dx$$

2. $$\int \frac{1}{x^2-5x+18}\,dx$$ (involves fractions and radicals)

3. $$\int \frac{1}{4x^2-12x+58}\,dx$$ (start by factoring out 4 from denominator)

4. $$\int \frac{1}{\sqrt{11+10x-x^2}}\,dx$$