

# CALCULUS 2 - REVIEW/PREVIEW UNIT 6

## INTEGRALS RESULTING IN INVERSE TRIG FUNCTIONS

BACKGROUND:  $\int \frac{1}{1+x^2} dx = \arctan x + C$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$$

$$\int \frac{1}{x\sqrt{x^2-1}} dx = \operatorname{arcsec} x + C$$

NOW WHAT IF WE CHANGE THE "1" TO ANOTHER CONSTANT?

EXAMPLE 1:  $\int \frac{1}{4+x^2} dx$  USE AN ALGEBRA STRATEGY!

$$= \int \frac{1}{4\left(1+\frac{x^2}{4}\right)} dx \quad \text{FACTOR OUT THE CONSTANT...}$$

$$= \frac{1}{4} \int \frac{1}{1+\left(\frac{x}{2}\right)^2} dx \quad \text{AND REARRANGE. NOW IT NEEDS A } u/du \text{ SUBSTITUTION.}$$

$$= \frac{1}{4} \cdot \int \frac{2}{1+u^2} du$$

$$\begin{cases} u = \frac{x}{2} \\ du = \frac{1}{2} dx \\ 2 du = dx \end{cases}$$

$$= \frac{1}{2} \int \frac{1}{1+u^2} du$$

$$= \frac{1}{2} \arctan(u) + C = \frac{1}{2} \arctan\left(\frac{x}{2}\right) + C$$

EXERCISE 1. USE THE ABOVE TECHNIQUE TO SHOW

$$\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$$

MANY CALC 2 STUDENTS THINK THIS FORMULA IS WORTH MEMORIZING.

EXAMPLE 2:  $\int \frac{1}{\sqrt{25-x^2}} dx$

$$= \int \frac{1}{\sqrt{25\left(1-\frac{x^2}{25}\right)}} dx$$

$$= \frac{1}{5} \int \frac{1}{\sqrt{1-\left(\frac{x}{5}\right)^2}} dx$$

$$\begin{cases} u = \frac{x}{5} \\ du = \frac{1}{5} dx \\ 5 du = dx \end{cases}$$

$$= \frac{1}{5} \int \frac{5}{\sqrt{1-u^2}} du$$

$$= \arcsin(u) + C = \arcsin\left(\frac{x}{5}\right) + C$$

EXERCISE 2: USE THE ABOVE TECHNIQUE TO SHOW

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin\left(\frac{x}{a}\right) + C$$

AGAIN, YOU MIGHT CHOOSE TO MEMORIZE THIS

EXAMPLE 3:

$$\int \frac{dx}{4x^2 + 36}$$

IT'S A SLIGHTLY DIFFERENT FORM.  
FIRST FACTOR OUT THE COEFFICIENT OF  $x^2$

$$= \frac{1}{4} \int \frac{dx}{x^2 + 9}$$

NOW JUST USE THE FORMULA FROM  
EXERCISE 2.

$$= \frac{1}{4} \int \frac{dx}{x^2 + 3^2}$$

$$= \frac{1}{4} \cdot \frac{1}{3} \arctan\left(\frac{x}{3}\right) + C$$

MORE EXERCISES: EVALUATE THESE INTEGRALS

3.  $\int \frac{1}{x\sqrt{x^2 - a^2}} dx$

4.  $\int \frac{1}{\sqrt{12 - x^2}} dx$

5.  $\int \frac{1}{x\sqrt{x^2 - 36}} dx$

6.  $\int \frac{3}{x^2 + 6} dx$

7.  $\int \frac{2}{7x^2 + 1} dx$