

CALC 2 - REVIEW/PREVIEW UNIT 3

ALGEBRA AND INTEGRALS

REMEMBER THAT IT IS OFTEN HELPFUL TO SIMPLIFY AN INTEGRAND BEFORE TRYING TO ANTI-DIFFERENTIATE.

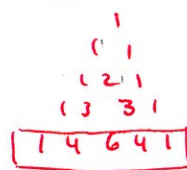
EXAMPLE 1: DISTRIBUTE

$$\begin{aligned} & \int \sqrt{x}(x-1) dx \\ &= \int x^{\frac{1}{2}}(x-1) dx \\ &= \int x^{\frac{3}{2}} - x^{\frac{1}{2}} dx \\ &= \frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} + C \end{aligned}$$

EXAMPLE 2: EXPAND

$$\begin{aligned} & \int (x^2+1)^4 dx \\ &= \int x^8 + 4x^6 + 6x^4 + 4x^2 + 1 dx \\ &= \frac{x^9}{9} + \frac{4}{7}x^7 + \frac{6}{5}x^5 + \frac{4}{3}x^3 + x + C \end{aligned}$$

PASCAL'S TRIANGLE:



These are the coefficients for the 4th degree binomial expansion

EXAMPLE 3: DISTRIBUTE

$$\begin{aligned} \int \frac{\sqrt{x}+1}{x} dx &= \int x^{-\frac{1}{2}} + \frac{1}{x} dx \\ &= 2x^{\frac{1}{2}} + \ln|x| + C \end{aligned}$$

EXAMPLE 4: FACTOR/CANCEL

$$\begin{aligned} & \int \frac{x^3+8}{x+2} dx \quad \text{numerator is the sum of two cubes} \\ &= \int \frac{(x+2)(x^2-2x+4)}{x+2} dx \\ &= \int x^2 - 2x + 4 dx \\ &= \frac{x^3}{3} - x^2 + 4x + C \end{aligned}$$

EXAMPLE 5: YOU FIGURE OUT THE TRICK!

$$\begin{aligned} & \int \frac{x-4}{\sqrt{x}+2} dx \quad \text{multiply and divide by conjugate} \\ &= \int \frac{(x-4)(\sqrt{x}-2)}{(\sqrt{x}+2)(\sqrt{x}-2)} dx \\ &= \int \frac{(x-4)(\sqrt{x}-2)}{x-4} dx \\ &= \int \sqrt{x}-2 dx = \frac{2}{3}x^{\frac{3}{2}} - 2x + C \end{aligned}$$

ALTERNATE TECHNIQUE: FACTOR NUMERATOR:

$$\int \frac{x-4}{\sqrt{x}+2} dx = \int \frac{(\sqrt{x}+2)(\sqrt{x}-2)}{\sqrt{x}+2} dx = \int \sqrt{x}-2 dx$$