Kempner Colloquium

On the behavior of sequences of compatible periodic orbits of particular prefractal billiard tables

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Mathematical billiards is a well-developed subfield of dynamical systems in which one attempts to understand how a pointmass traverses some region (possibly) subject to collisions in a sufficiently smooth boundary or obstruction.

Classical examples of mathematical billiard tables include the unit square, ellipse, horseshoe, mushroom and so on. In this talk, we will initially focus on the behavior of a pointmass as it traverses the equilateral triangle billiard $\Omega(\Delta)$ and the unit square billiard $\Omega(Q)$. In general, the billiard dynamics on what is called a *rational polgyonal billiard table* are very well understood. We examine the billiard dynamics on a prefractal billiard table $\Omega(F_n)$, where $\{F_n\}_{n=0}^{\infty}$ is a suitable sequence of prefractal billiards converging in the Hausdorff metric to a fractal F, where F is either the Koch snowflake KS, a Sierpinski carpet $S_{\mathbf{a}}$ or the T-fractal $\mathscr{T}$. We draw upon an established literature when constructing what we are calling a *sequence of compatible orbits*. We then examine the behavior of a sequence of compatible periodic orbits of prefractal billiard tables F_n, where, for every $n \geq 0$, $F_n = KS_n$, $S_{\mathbf{a},n}$ or $\mathscr{T}_n$. In so doing, we determine periodic orbits of $\Omega(KS$, $\Omega(S_{\mathbf{a}})$ and $\Omega(\mathscr{T})$. In addition to this, in the case of $\Omega(KS)$, we determine a sequence of line segments constituting what we are calling a *nontrivial path*. Such a path connects two elusive points of the Koch snowflake. We then indicate how such paths should also exist in $\Omega(\mathscr{T})$.

The *eventual* goal of this research is to determine a well-defined phase space for the billiard dynamics on a fractal billiard table. Doing so may require an alternate approach than that presented here. We briefly outline other possible approaches and end our talk by indicating directions for future research.

December 10, 2012
4:00 p.m.
MATH 350