Kempner Colloquium

Cohen-Lenstra heuristics for function fields and Hurwitz spaces

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The Cohen-Lenstra heuristics are a remarkable set of conjectures made by Henri Cohen and Hendryk Lenstra in the 80's about the distribution of class groups of number fields. Here, the class group Cl_K of a number field K is a finite abelian group, and captures some of the arithmetic of K. For instance, it is trivial if and only if unique factorization holds in the ring of integers $\mathcal{O}_K \subseteq K$.

Loosely speaking, Cohen and Lenstra predict that a particular finite abelian group A occurs as the class group of a quadratic, imaginary number field $K = \mathbb{Q}[\sqrt{-d}]$ with probability inversely proportional to the size of the automorphism group $Aut(A)$. These heuristics remain unproven, but are supported by a weight of computational evidence. Indeed, it appears that this sort of evidence was part of Cohen and Lenstra's original justification in forming this conjecture.

In joint work with Jordan Ellenberg and Akshay Venkatesh, however, we have proven an analogue of these heuristics in the setting of function fields – where $\mathbb{Q}$ is, for instance, replaced by the field of rational functions $\mathbb{F}_q(t)$ over the finite field $\mathbb{F}_q$.

In this talk, I'll give an introduction to the heuristics, explain how algebraic geometry allows us to translate the function field conjectures into a question about moduli spaces of branched covers, and how algebraic topology can be used to answer that question.

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MATH 220