Kempner Colloquium

# Contractions of Lie Groups and Representation Theory.

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Let $K$ be a closed subgroup of a Lie group G. The contraction of $G$ to $K$ is a Lie group, usually more elementary in structure than $G$ itself, that approximates $G$ to first order near K. The terminology is due to the mathematical physicists, who examined the group of Galilean transformations as a contraction of the group of Lorentz transformations. My focus will be on a related but different class of examples, the prototype of which is the group of isometric motions of Euclidean space, viewed as a contraction of the group of isometric motions of hyperbolic space.

It is natural to expect some sort of limiting relation between representations of the contraction and representations of $G$. But in the 1970s George Mackey carried out a few calculations pointing to an interesting rigidity phenomenon: as the contraction group is deformed back to $G$, the representation theory remains in some sense unchanged. In particular the irreducible representations of the contraction group parametrize the irreducible representations of $G$. I shall formulate a reasonably precise conjecture that was inspired by subsequent developments in $C^*$-algebra theory and noncommutative geometry, and describe the evidence in support of it, which is by now substantial. However a conceptual explanation of Mackey's rigidity phenomenon remains elusive.

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