Kempner Colloquium

On the number of embeddings of a graph into another

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The sequence $X_n = Cay(\mathcal{Z}_n, \mathcal{U}_n)$ (where $\mathcal{Z}_n$ is the ring of congruences modulo n and U_n the multiplicative group of invertible elements of the ring) is called the sequence of Unitary Cayley Graphs. Quite a few years ago I noticed that the number of triangles in X_n, denoted by $p_3(n)$ satisfies $6p_3(n) = n^3 \prod_{p/n}(1-1/p)(1-2/p)$. Where the product is taken over the prime divisors p of n. The right hand side of the equation is a multiplicative function. Next, in [B-G] we proved that for each $k \geq 3$, the function $2kp_k(n)$ is a $\mathcal{Z}$-linear combination of multiplicative functions, where $p_k(n)$ is the number of k-cycles of X_n, explicit formulas were given for $k = 4$. Deivi Luzardo in his Ph. D Thesis proved that if $F_H(G)$ counts the number of embeddings of the graph H in the graph G. Then F_H is a $\mathcal{Z}$-linear combination of functions which are multiplicative under the Conjunction of Graphs. We will explain Deivi's result and show how to obtain from it the previous ones. We will also discuss a recent related problem.

September 24, 2012
4:00 p.m.
MATH 350