Kempner Colloquium

Three-wave resonant interactions

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A resonant interaction of three wavetrains is the simplest nonlinear and non-trivial interaction of dispersive waves, propagating in a medium without dissipation. Coppi, Rosenbluth & Sudan (1969) noted that this kind of interaction takes two forms: the decay case, where energy is conserved, and the explosive case, in which almost all solutions blow up in finite time. Either way, the partial differential equations that describe these processes were shown to be completely integrable by Zakharov & Manakov (1973), and solutions of the problem for spatially localized wave packets were given by Zakharov & Manakov (1976), Kaup (1976) and others. Numerical simulations of the process usually impose periodic boundary conditions, and the known methods of analytical solution fail with these boundary conditions.

We present an alternative way to study this problem, in terms of convergent Laurent series (in"time"), which contain five, real-valued functions (in "space"). These functions must obey some differentiability constraints, but are otherwise arbitrary – they can be periodic, or almost periodic, or localized in space. A general solution of the problem would involve six such functions, so our current work stops short of a general solution. For simplicity, we work in one spatial dimension, and we analyze only the explosive case.

Feburary 10, 2014
4:00 p.m.
MATH 350