ABSTRACTS: NCG FESTIVAL BOULDER 2025

Therese Basa Landry, University of California, Santa Barbara, Quantum Metrics on Quantum Compact Groups

We investigate quantum compact groups which support quantum metric space structure. In our core example, we define an analog of the Hamming metric on the quantum permutation group S_n^+ . We also obtain an associated quantum 1-Wasserstein distance on the tracial state space of $C(S_n^+)$. This is joint work with David Jekel and Anshu Nirbay.

Rachel Chaiser, Kalamazoo College,

A Low-Dimensional Counterexample to the HK-conjecture

Matui's HK-conjecture proposes an in-principle computation of the K-theory of the reduced C*-algebra of certain groupoids in terms of the homology of the groupoid. While there are a number of positive results, the conjecture fails in general with early counterexamples due to Scarparo and to Deeley. Â Deeley gave the first counterexample where the groupoid is principal by building an odometer from a flat manifold of dimension 9 and an expansive self-cover. For these odometers, the invariants in the HK-conjecture are computed from the corresponding invariants of the flat manifold, and the HK-conjecture is a direct analogue of the following phenomenon in topology. A The Atiyah–Hirzebruch spectral sequence computes the K-theory of a CW-complex of dimension at most 3 explicitly as a direct sum of its cohomology groups; but this computation fails in general in dimensions 4 and greater. In analogy with the topological situation, I will present a counterexample to the HK-conjecture coming from a flat manifold of dimension 4.Å I extend this result to dimensions greater than 4 and show that the HK-conjecture holds if the manifold has dimension 3 or less.

Alonso Delfin Ares de Parga, CU Boulder,

Equivalence Theorem for (reduced) Banach algebras associated with étale groupoid

Renault's Equivalence Theorem roughly states that an equivalence of groupoids G and H induces a Morita equivalence between their full C*-algebras C * (G) and C * (H). Later, A. Sims and D. P. Williams formally obtained this result for reduced groupoid C*-algebras. Around the same time, W. Paravicini proved that the theorem also holds for any unconditional completion A(G) of $C_c(G)$.

In this talk, I will go over the more general notion of Morita equivalence for Banach algebras. Then I will show how to get $F_r^p(G)$, the reduced L^p -operator algebra of an étale groupoid G, and explain why Renault's Equivalence Theorem remains valid in this case, even though $F_r^p(G)$ is not generally an unconditional completions of $C_c(G)$.

If time permits, I will also discuss some of the issues in deciding whether Renault's theorem holds for the full L^p -groupoid algebras, which currently remains an open question.

Arturo Jaime, University of Hawaii,

Controlled KK-theory, index pairing, and the closure of zero.

There is a canonical, possibly non-Hausdorff, topology on $KK_0(A, B)$ endowing it with the structure of a topological group. Brown-Douglas-Filmore first considered a topology on the Ext-theory groups, and Brown showed that the closure of zero in the Ext group corresponds precisely with quasidiagonal extensions. Further studied by Salinas and Schochet in the setting of KK-theory, it turns out that in the case that A is a nuclear C^* -algebra that satisfies the UCT, then the closure of zero in $KK_0(A, B)$ corresponds with the Pext subgroup of $\operatorname{Ext}_{\mathbb{Z}}^1(K_*(A), K_{*+1}(B))$. Using the controlled picture of KK-theory introduced by Willett and Yu, I will speak on the construction of how for any general pair of separable C^* -algebras (A, B), one can show that the closure of zero sits inside the Pext subgroup using a "controlled" index pairing. This is work in progress.

Sheagan John, CU Boulder,

Localized Versions of Cyclic Homology and Cohomology

Inspired by the localized K-theory construction of Moscovici and Wu, we define localized versions of cyclic homology and cohomology. It is then possible to consider construction of a localized bivariant Connes-Chern character from localized K-theory (K-homology) to localized cyclic homology (cohomology). This is joint work with Markus Pflaum.

Levi Lorenzo, CU Boulder,

Finitely Summable K-Homology, The Index Pairing, and Cantor Minimal Systems

K-theory is the invariant that has proven incredibly effective at classifying C^* - algebras. In this talk, we discuss its dual theory, K-homology. K-homology and K-theory admit a bilinear pairing to the integers, called the index pairing, which allows one to learn about K-theory using K-homology and vice versa. In general, this pairing is very difficult to compute; however, if a C*algebra admits a dense subalgebra on which its K-homology is finitely summable, Connes equips us with more computable formulas. Given a smooth manifold, M, of dimension n, the Khomology of M is finitely summable on $C^{\infty}(M)$ for p > n. We can ask whether similar results hold in the noncommutative setting. For example, Connes showed that the irrational rotation algebras have uniformly finitely summable K-homology. The irrational rotation algebras are crossed product algebras arising from minimal actions on the circle. In my work, I examine the case of the Cantor Set and ask whether the K-homology of crossed products arising from minimal actions on it can be taken to be uniformly finitely summable. While we do not find a dense subalgebra on which the K-Homology of a general Cantor Minimal Systems can be taken to be uniformly summable, we utilize Putnam's orbit-breaking AF subalgebras to produce cycles through which we can compute any index pairing using Connes' Trace Formulas.

Maggie Reardon, CU Boulder,

The HK-conjecture for certain groupoids constructed by Putnam

Matui's HK-conjecture proposes a relationship between the homology groups of a nice enough groupoid and the K-theory of the associated reduced C^* -algebra. The conjecture is not true in general and there are a number of counterexamples. However, the conjecture holds for certain classes of groupoids including AF groupoids.

Putnam introduced a new class of groupoids in the paper titled "Some classifiable groupoid C^* -algebras with prescribed K-theory". These new groupoids are related to AF groupoids and this prompts a natural question: does this new class of groupoids satisfy the HK-conjecture?

Jesus Sanchez Jr., University of Washington St Louis, Hypoelliptic Operators on Contact Manifolds

In this talk we will provide a construction of a new hypoelliptic operator on the Heisenberg group and discuss its properties and generalizations to contact manifolds. This is joint work with Andres Franco Valiente.

Xiaoyu Su, Texas A&M,

Finite dynamical complexity and quantitative stable ranks

Finite dynamical complexity for dynamical systems, introduced by Guentner, Willett, and Yu in 2016, extends the geometric concept of finite decomposition complexity for metric spaces (developed by the first and third authors with Tessera) and builds upon the notion of dynamic asymptotic dimension for dynamical systems (introduced by same three authors). This framework captures the ability to iteratively decompose a dynamical system into smaller, more manageable components. Inspired by this idea, we introduce the concept of quantitative stable rank, a quantitative analogue of the classical stable rank, and use it to establish bounds on the classical stable rank of the C^* -algebra associated with dynamical systems exhibiting finite dynamical complexity.

Hao Zhuang, University of Washington St Louis,

Kervaire semi-characteristics in KK-theory

In 1969, Atiyah and Singer proposed the concept of mod 2 index. Later, for any closed oriented (4n+1)-manifold, Atiyah identified its Kervaire semi-characteristic with the mod 2 index of a skew-adjoint operator. By perturbing the operator without changing its mod 2 index, Atiyah proved a vanishing theorem of the Kervaire semi-characteristic.

We hope to find the counterpart of Atiyah and Singer's work on noncompact manifolds. Thus, we construct and explore a semi-characteristic for noncompact manifolds admitting proper cocompact Lie group actions. The construction is both analytic via KK-theory and topological via deformed harmonic forms. In particular, this semi-characteristic admits an Atiyah type vanishing theorem. The work is based on Bunke, Mathai, and Zhang's proper cocompact elliptic theory and Tang, Yao, and Zhang's proper compact Hodge theorem.