Solutions: 1/17/18

Warm-Up:

1. Evaluate the following integrals

   (a) $\int x^2(\sqrt{x}+5)+e^2\,dx$ $= \int x^{5/2} + 5x^2 + e^2\,dx$

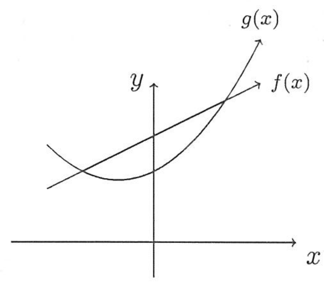
   $= \frac{2}{7}x^{7/2} + \frac{5}{3}x^3 + e^2x + C$

   (b) $\int_1^2 \frac{3x^3+1}{4x}\,dx$ $= \int_1^2 \frac{3}{4}x^2 + \frac{1}{4}\cdot\frac{1}{x}\,dx = \left[\frac{1}{4}x^3 + \frac{1}{4}\ln|x|\right]_1^2$

   $= \left[\frac{1}{4}\cdot 8 + \frac{1}{4}\ln(2)\right] - \left[\frac{1}{4}\cdot 1 + \frac{1}{4}\ln(1)\right]$

   $= 2 + \frac{1}{4}\ln(2) - \frac{1}{4} + 0$

2. Find the area bounded between the line $f(x) = x+3$ and the parabola $g(x) = x^2+x+2$.



   (a) Find where $f(x)$ and $g(x)$ intersect by setting them equal and solving for x.

   $x+3 = x^2+x+2$

   $0 = x^2-1$

   $0 = (x+1)(x-1)$

   $f(x), g(x)$ intersect when $x = \pm 1$

   (b) Set up the integral and evaluate to find the area bounded by $f(x)$ and $g(x)$.

   $\int_{-1}^{1} f(x) - g(x)\,dx = \int_{-1}^{1} (x+3) - (x^2+x+2)\,dx$

   $= \int_{-1}^{1} -x^2+1\,dx$

   $= \left[-\frac{1}{3}x^3 + x\right]_{-1}^{1}$

   $= \left[-\frac{1}{3}+1\right] - \left[\frac{1}{3}-1\right]$

   $= \frac{2}{3} - -\frac{2}{3}$

   $= 4/3$

3. Evaluate the following indefinite integrals using substitution.

(a) $\displaystyle\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx$ $= \int 2\cos(u)\,du = 2\sin(u) + C = 2\sin(\sqrt{x}) + C$

$\left\{ \begin{array}{l} u = \sqrt{x} = x^{1/2} \\ du = \frac{1}{2}x^{-1/2}\,dx \\ 2\,du = \frac{1}{\sqrt{x}}\,dx \end{array} \right\}$

(b) $\displaystyle\int \frac{e^x}{e^x+1}\,dx$ $= \int \frac{1}{u}\,du = \ln|u| + C = \ln|e^x+1| + C$

$\left\{ \begin{array}{l} u = e^x + 1 \\ du = e^x\,dx \end{array} \right\}$

4. Evaluate the following definite integrals using substitution.

(a) $\displaystyle\int_{x=2}^{x=3} \frac{xe^{x^2}}{3}\,dx$ $= \displaystyle\int_{u=4}^{u=9} \frac{e^u}{3}\cdot\frac{1}{2}\,du = \frac{1}{6}\int_4^9 e^u\,du = \frac{1}{6}e^u\Big|_4^9$

$= \frac{1}{6}e^9 - \frac{1}{6}e^4$

$\left\{ \begin{array}{l} u = x^2 \\ du = 2x\,dx \\ \frac{1}{2}du = x\,dx \end{array} \right\}$

(b) $\displaystyle\int_{x=0}^{x=1} \frac{x}{1+3x^2}\,dx$ $= \displaystyle\int_{u=1}^{u=4} \frac{1}{6}\frac{1}{u}\,du = \frac{1}{6}\ln|u|\Big|_1^4 = \frac{1}{6}\ln|4| - \frac{1}{6}\ln|1|$

$= \frac{1}{6}\ln(4) - 0$

$\left\{ \begin{array}{l} u = 1+3x^2 \\ du = 6x\,dx \\ \frac{1}{6}du = x\,dx \end{array} \right\}$

5. Show the following two integrals are equivalent:

$$\int_0^2 3x\sqrt{9-x^2}\,dx = \int_5^9 \frac{3\sqrt{u}}{2}\,du.$$

$\displaystyle\int_{x=0}^{x=2} 3x\sqrt{9-x^2}\,dx = \int_{u=9}^{u=5} -\frac{1}{2}\cdot 3\sqrt{u}\,du = -\int_9^5 \frac{3\sqrt{u}}{2}\,du = \int_5^9 \frac{3\sqrt{u}}{2}\,du$

$\left\{ \begin{array}{l} u = 9-x^2 \\ du = -2x\,dx \\ -\frac{1}{2}du = x\,dx \end{array} \right\}$

switching limits changes sign in front