Goal: To identify what (if any) u-substitutions are necessary to compute an integral and to practice making such substitutions.

For each problem, identify what (if any) u-substitution(s) need to be made to evaluate each integral. Make the substitution and simplify, but **do not** evaluate the integral.

1. $\displaystyle\int x\sin(x^2)\,dx$

   Solution: $u = x^2, du = 2x\,dx,$
   $$\int x\sin(x^2)\,dx = \frac{1}{2}\int \sin(u)\,du$$

2. $\displaystyle\int \sqrt{x}(x+3)\,dx$

   Solution: No substitution needed,
   $$\int \sqrt{x}(x+3)\,dx = \int x^{3/2} + 3x^{1/2}\,dx$$

3. $\displaystyle\int x\sqrt{x+3}\,dx$

   Solution: $u = x+3, du = \ dx,$
   $$\int x\sqrt{x+3}\,dx = \int (u-3)\sqrt{u}\,du = \int u^{3/2} - 3u^{1/2}\,du$$

4. $\displaystyle\int \frac{\sqrt{\ln(x)}}{x}\,dx$

   Solution: $u = \ln(x), du = \frac{1}{x}\,dx,$
   $$\int \frac{\sqrt{\ln(x)}}{x}\,dx = \int \sqrt{u}\,du$$

5. $\int \frac{x+4}{x}\,dx$

Solution: No substitution needed,
$\int \frac{x+4}{x}\,dx = \int 1 + \frac{4}{x}\,dx$

6. $\int \frac{x}{x+4}\,dx$

Solution: $u = x+4, du = \ dx,$
$\int \frac{x}{x+4}\,dx = \int \frac{u-4}{u}\,du = \int 1 - \frac{4}{u}\,du$

7. $\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

Solution: $u = e^x, du = e^x\,dx,$
$\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx = \int \frac{1}{\sqrt{1-u^2}}\,du$

8. $\int \frac{\arctan(x)}{1+x^2}\,dx$

Solution: $u = \arctan(x), du = \frac{1}{1+x^2}\,dx,$
$\int \frac{\arctan(x)}{1+x^2}\,dx = \int u\,du$

9. $\displaystyle\int \frac{x^3}{(1+x^2)^2}\,dx$

Solution: $u = 1 + x^2, du = 2x\,dx,$
$\displaystyle\int \frac{x^3}{(1+x^2)^2}\,dx = \frac{1}{2}\int \frac{x^2}{(1+x^2)^2}\cdot 2x\,dx = \frac{1}{2}\int \frac{u-1}{u^2}\,du = \frac{1}{2}\int \frac{1}{u} - \frac{1}{u^2}\,du$

10. $\displaystyle\int \frac{x^2}{\sqrt{1-x^3}}\,dx$

Solution: $u = 1 - x^3, du = -3x^2\,dx,$
$\displaystyle\int \frac{x^2}{\sqrt{1-x^3}}\,dx = -\frac{1}{3}\int \frac{1}{\sqrt{u}}\,du$

11. $\displaystyle\int \sin(x)(3\cos^4(x) + 4\cos^3(x) - 9)\,dx$

Solution: $u = \cos(x), du = -\sin(x)\,dx,$
$\displaystyle\int \sin(x)(3\cos^4(x) + 4\cos^3(x) - 9)\,dx = -\int 3u^4 + 4u^3 - 9\,du$

12. $\displaystyle\int x^3 + e^{3-x}\,dx$

Solution: Split into two integrals. The first doesn't need a substitution.
For the second, use $u = 3 - x, du = -\,dx,$
$\displaystyle\int x^3 + e^{3-x}\,dx = \int x^3\,dx - \int e^u\,du$

13. $\displaystyle\int \frac{\sin(\sqrt{x}+1)}{\sqrt{x}} + \frac{1}{x^2+1}\,dx$

Solution: Again split into two integrals.
For the first, use $u = \sqrt{x} + 1, du = \frac{1}{2\sqrt{x}}\,dx,$
The second integral doesn't need a substitution.
$\displaystyle\int \frac{\sin(\sqrt{x}+1)}{\sqrt{x}} + \frac{1}{x^2+1}\,dx = 2\int \sin(u)\,du + \int \frac{1}{x^2+1}\,dx$