Solutions: 1/16/18

Below is a list of indefinite integrals that you know how to find from your Calculus I class. Evaluate these integrals.

1. $\int x^2\,dx$ $= \frac{1}{3}x^3 + C$
2. $\int x^n\,dx$ for $n \neq -1$ $= \frac{1}{n+1}x^{n+1} + C$
3. $\int \frac{1}{a}\,da$ $= \ln|a| + C$
4. $\int e^\alpha\,d\alpha$ $= e^\alpha + C$
5. $\int 7^q\,dq$ $= \frac{1}{\ln(7)}7^q + C$
6. $\int b^x\,dx$ for $b > 0$ $= \frac{1}{\ln(b)}b^x + C$
7. $\int \sin\theta\,d\theta$ $= -\cos\theta + C$
8. $\int \cos\phi\,d\phi$ $= \sin\phi + C$
9. $\int \tan w\,dw = \int \frac{\sin w}{\cos w}\,dw \left\{\begin{array}{l} u = \cos w \\ du = -\sin w\,dw \\ -du = \sin w\,dw \end{array}\right\} = \int \frac{-1}{u}\,du = -\ln|u| + C = -\ln|\cos w| + C$
10. $\int \cot v\,dv = \int \frac{\cos v}{\sin v}\,dv \left\{\begin{array}{l} u = \sin v \\ du = \cos v\,dv \end{array}\right\} = \int \frac{1}{u}\,du = \ln|u| + C = \ln|\sin v| + C$
11. $\int \sec^2 u\,du$ $= \tan u + C$
12. $\int \sec y \tan y\,dy$ $= \sec y + C$
13. $\int \csc^2\beta\,d\beta$ $= -\cot\beta + C$
14. $\int \csc r \cot r\,dr$ $= -\csc r + C$
15. $\int \frac{1}{1+\xi^2}\,d\xi$ $= \arctan(\xi) + C$
16. $\int \frac{1}{\sqrt{1-\ell^2}}\,d\ell$ $= \arcsin(\ell) + C$

17. $\int \sqrt{z}(z^2+6z+4)\,dz$ $= \int z^{5/2} + 6z^{3/2} + 4z^{1/2}\,dz = \frac{2}{7}z^{7/2} + \frac{12}{5}z^{5/2} + \frac{8}{3}z^{3/2} + C$

18. $\int \frac{6y^8+12y^2-y^6}{3y^7}\,dy$ $= \int 2y + 4y^{-5} - \frac{1}{3}\cdot\frac{1}{y}\,dy = y^2 - y^{-4} - \frac{1}{3}\ln|y| + C$

19. $\int 5e^m + \cos m + m^3\,dm$ $= 5e^m + \sin(m) + \frac{1}{4}m^4 + C$

20. $\int \frac{2b}{b^2+7}\,db$ $= \int \frac{1}{u}\,du = \ln|u| + C = \ln|b^2+7| + C$

$\begin{cases} u = b^2+7 \\ du = 2b\,db \end{cases}$

21. (a) $\int \frac{\sec^2 x + \sec x\tan x}{\sec x + \tan x}\,dx$ $= \int \frac{1}{u}\,du = \ln|u| + C = \ln|\sec x + \tan x| + C$

$\begin{cases} u = \sec x + \tan x \\ du = (\sec x\tan x + \sec^2 x)\,dx \end{cases}$

(b) $\int \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x}\,dx$ $=$ same as above $= \ln|\sec x + \tan x| + C$

(c) $\int \sec x\,dx$ $=$ same as above $= \ln|\sec x + \tan x| + C$

22. $\int \csc x\,dx = \int \frac{\csc x(\csc x + \cot x)}{\csc x + \cot x}\,dx = \int \frac{\csc^2 x + \csc x\cdot\cot x}{\csc x + \cot x}\,dx$

$\begin{cases} u = \csc x + \cot x \\ du = (-\csc x\cot x - \csc^2 x)\,dx \end{cases}$

$= -\int \frac{1}{u}\,du$

$= -\ln|u| + C$

$= -\ln|\csc x + \cot x| + C$