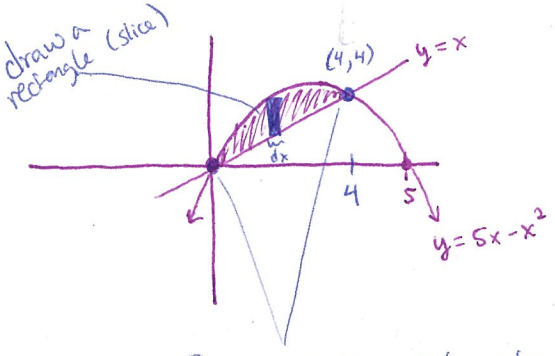
Math 2300-007: §6.1 - More About Areas

(With thanks to Faan Tone Liu) Solutions 2/7/18

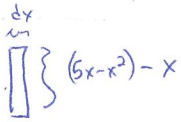
Complete the following problems about areas. As you work, think about the key points, so can summarize them in question 4.

1. Find the area bounded between $y = 5x - x^2$ and $y = x$.



$y = x(5-x)$
zeros: $x=0,5$

zero: $x=0$



To find these intersections, set the equations equal to each other:

$5x - x^2 = x$

$0 = x^2 - 4x$

$0 = x(x-4)$

$x=0,\ x=4$

$$\text{Area} = \int_{x=0}^{x=4} \text{Area of small rectangle}$$

$$= \int_0^4 \text{height} \cdot dx$$

$$= \int_0^4 (5x-x^2) - x\,dx$$

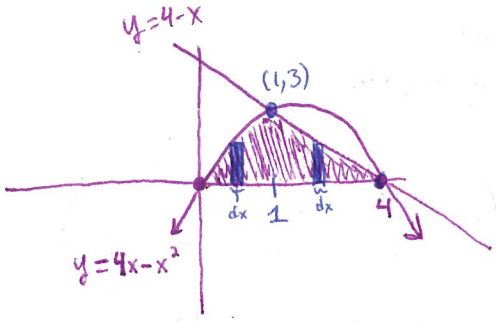
$$= \int_0^4 4x - x^2\,dx$$

$$= \left[2x^2 - \tfrac{1}{3}x^3\right]_0^4$$

$$= (2\cdot 16 - \tfrac{1}{3}64) - (0-0)$$

$$= \boxed{32 - \frac{64}{3}}$$

2. Find the area of the region bounded by $y = 4x - x^2$, $y = 4 - x$, and the x-axis.



$y = x(4-x)$
zeros: $x=0,4$

zero: $x=4$

⚠ Here, the top curve changes as the slice moves from left to right. Fix: do two integrals:

To find intersections:

$4x - x^2 = 4 - x$

$0 = x^2 - 5x + 4$

$0 = (x-1)(x-4)$

$x=1,\ x=4$

$$\text{Area} = \int_0^1 [(4x-x^2) - 0]\,dx + \int_1^4 [(4-x) - 0]\,dx$$

$$= \int_0^1 4x - x^2\,dx + \int_1^4 4 - x\,dx$$

$$= \left[2x^2 - \tfrac{1}{3}x^3\right]_0^1 + \left[4x - \tfrac{1}{2}x^2\right]_1^4$$

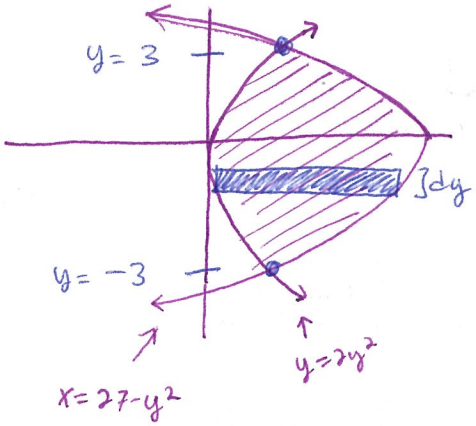
$$= \left(2 - \tfrac{1}{3}\right) - (0-0) + (16-8) - \left(4 - \tfrac{1}{2}\right)$$

$$= \frac{5}{3} + 8 - \frac{7}{2}$$

$$= \frac{10 + 48 - 21}{6} = \boxed{\frac{37}{6}}$$

3. Set up the integral to find the area bounded between $x = 2y^2$ and $x = 27 - y^2$.

Typo!



⚠ Use a "dy" integral.
horizontal ▭ dy
rectangles

▭ dy
width

$$\text{Area} = \sum_{y=-3}^{y=3} \text{Area of slice} = \int_{-3}^{3} \text{width} \cdot dy$$

$$= \int_{-3}^{3} (\text{right} - \text{left})\, dy = \int_{-3}^{3} (27 - y^2) - 2y^2\, dy$$

$$= \int_{-3}^{3} 27 - 3y^2\, dy = \left[27y - y^3\right]_{-3}^{3}$$

$$= (81 - 27) - (-81 + 27)$$

$$= 2(81 - 27) = 2(54) = \boxed{108}$$

Intersections:

$$x_1 = x_2$$
$$2y^2 = 27 - y^2$$
$$3y^2 = 27$$
$$y^2 = 9$$
$$y = \pm 3$$

4. What advice would you give to another student for finding areas between curves? What are some of the steps? The key points?

* Draw a Picture!
  - ↳ To find intersections, set the equations equal
  - ↳ it helps to include x- & y-intercepts, intersections, etc...
  - ↳ Draw a small rectangle

▯ dx OR ▭ dy (Choose dx, dy based on which is easier to you.)

* Use an integral to "add up" the areas of the rectangles
  - ↳ for the height of ▯ dx, do "top function − bottom function"
  - ↳ for the width of ▭ dy, do "right function − left function"

* May have to use multiple integrals (see #2).