Math 2300-013: Quiz 11

Name: Solution Score: ________

1. Solve the differential equation

$$\frac{du}{dt} = \frac{3}{ut^2} + \frac{\sec^2(t)}{2u}$$

subject to the initial condition $u(\pi/4) = 1$.

$$\frac{du}{dt} = \left(\frac{3}{t^2} + \frac{\sec^2(t)}{2}\right)\frac{1}{u}$$

$$u\,du = 3t^{-2} + \tfrac{1}{2}\sec^2(t)\,dt$$

$$\int u\,du = \int 3t^{-2} + \tfrac{1}{2}\sec^2(t)\,dt$$

$$\tfrac{1}{2}u^2 = \frac{-3}{t} + \tfrac{1}{2}\tan(t) + C \longrightarrow$$

Solve for C:

$$u(\pi/4) = 1$$

$$\tfrac{1}{2}\cdot 1^2 = \frac{-3}{\pi/4} + \tfrac{1}{2}\cdot\frac{\sin(\pi/4)}{\cos(\pi/4)} + C$$

$$\tfrac{1}{2} = \frac{-12}{\pi} + \tfrac{1}{2}\cdot 1 + C$$

$$C = \frac{12}{\pi}$$

$$\tfrac{1}{2}u^2 = \frac{-3}{t} + \tfrac{1}{2}\tan(t) + \frac{12}{\pi}$$

$$u^2 = -\frac{6}{t} + \tan(t) + \frac{24}{\pi}$$

$$u = \pm\sqrt{\frac{-6}{t} + \tan t + \frac{24}{\pi}}$$

by initial condition, $u(\pi/4)=1$, we know $u \geq 0$.

$$u = \sqrt{\frac{-6}{t} + \tan t + \frac{24}{\pi}}$$

2. Suppose $P(t)$ represents the size of a population in millions t years since 2000 and we know that

- the birth rate is 0.05 births per person per year;
- the death rate is 0.02 deaths per person per year;
- 3 million immigrants join the population each year.

Write (**but do not solve**) a differential equation for $\frac{dP}{dt}$, the rate of change of the population at time t.

$$\frac{dP}{dt} = \underbrace{0.05 \cdot P}_{\text{population in}} - \underbrace{0.02 \cdot P}_{\text{population out}} + 3$$

also part of population in.

$$\boxed{\frac{dP}{dt} = 0.03P + 3}$$