1. A salt brine tank has pure water flowing in at 10 L/min. The contents of the tank are mixed thoroughly and continuously. The brine flows out at 10 L/min. Initially, the tank contains 150 L of brine, at a concentration of 5 g/L. Follow the steps below to determine the concentration of brine after 30 minutes, and the limiting concentration of the brine.

   (a) Let $S(t)$ = amount of salt in the tank at time $t$ (in g)
   and let $C(t)$ = concentration of salt in the tank at time $t$ (in g/L)
   $C(0) =$ 5 g/L

   $S(0) =$ 5 g/L ×150L = 750g

   Write $C(t)$ in terms of $S(t)$:
   $C(t) = \frac{S(t)}{150}\frac{\text{g of salt}}{\text{L}}$

   (b) Now write a differential equation describing how fast is the salt is leaving the tank.

   **Solution:** $\frac{dS}{dt} = C(t)\frac{\text{g of salt}}{\text{L}} \times -10\frac{\text{L}}{\text{min}} = -\frac{S}{150} \cdot 10\frac{\text{g of salt}}{\text{min}} = -\frac{S}{15}\frac{\text{g of salt}}{\text{min}}$

   (c) Solve the initial value problem $\frac{dS}{dt} = -\frac{S}{15}$, $S(0) = 750$.

   **Solution:** Separate variables:
   $\int \frac{dS}{S} = -\int \frac{1}{15}dt$

   $\ln|S| = -\frac{1}{15}t + C$

   $S = Ae^{-\frac{t}{15}}$

   Substitute $t = 0$, $S = 750$:

   $S = 750e^{-\frac{t}{15}}$

   (d) What is the concentration when $t = 30$?

   **Solution:** $S(30) = 750e^{-2}$, so $C(30) = \frac{750e^{-2}}{150} = \frac{5}{e^2}$

   (e) What is the limiting concentration of the brine?

   **Solution:** $\lim_{t\to\infty} 750e^{-\frac{t}{15}} = 0$. This makes sense because we are adding pure water into the brine, so it gets increasingly dilute. In the limit, there is nothing left.

2. As before, a salt brine tank contains 150 L of brine at a concentration of 5 g/L. But this time brine at a concentration of 2g/L is pumped into the tank at a rate of 10 L/min. The contents of the tank are mixed thoroughly and continuously and the brine flows out at 10 L/min. Follow the steps below to determine how long until the concentration is 3 g/L, and what the limiting concentration is.

   (a) Again, let $S(t)$ = amount of salt in the tank at time $t$ (in g)
   and let $C(t)$ = concentration of salt in the tank at time $t$ (in g/L)
   $C(0) =$ 5 g/L

   $S(0) =$ 5 g/L $\times$150L = 750g

   Write $C(t)$ in terms of $S(t)$:
   $C(t) = \frac{S(t)}{150}\frac{\text{g}}{\text{L}}$

   (b) How fast is salt entering the tank?

   **Solution:** $2\frac{\text{g}}{\text{L}} \times 10\frac{\text{L}}{\text{min}} = 20\frac{\text{g}}{\text{min}}$

   (c) How fast is salt leaving the tank?

   **Solution:** $C\frac{\text{g}}{\text{L}} \times 10\frac{\text{L}}{\text{min}} = \frac{S}{150} \times 10\frac{\text{g}}{\text{min}} = \frac{S}{15}\frac{\text{g}}{\text{min}}$

   (d) What is the net change of the salt in the tank, $\frac{dS}{dt}$?

   **Solution:** $\frac{dS}{dt} = 20 - \frac{S}{15}$

   (e) Solve the initial value problem $\frac{dS}{dt} = 20 - \frac{S}{15}$, $S(0) = 750$.

   **Solution:** Separate variables:
   $\int \frac{1}{20 - \frac{S}{15}} dS = \int dt$
   $\int \frac{1}{300 - S} dS = \int \frac{1}{15} dt$
   $\int \frac{1}{S - 300} dS = \int -\frac{1}{15} dt$
   $\ln|S - 300| = -\frac{1}{15}t + C$
   $S - 300 = Ae^{-\frac{1}{15}t}$
   Now substitute $t = 0$, $S = 750$:
   $A = 450$
   $S = 450e^{-\frac{1}{15}t} + 300$ g

   (f) When is $C(t) = 3$ g/L? What is the limiting concentration?

   **Solution:** We are looking for when $S(t) = 450$.
   $150 = 450e^{-\frac{1}{15}t}$, giving $\frac{1}{3} = e^{-\frac{1}{15}t}$, solving for $t$ gives $t = 15\ln 3$.
   Now $\lim_{t\to\infty}(450e^{-\frac{1}{15}t} + 300) = 300$. So the limiting concentration is $C = \frac{300}{150} = 2$ g/L.