§7.3: Separable Differential Equations

Key Points:

- A **separable differential equation** is a differential equation that can be written in the form
$$f(y) \cdot \frac{dy}{dx} = g(x).$$

- To solve a separable differential equation:

  1. Separate the variables
  2. Integrate both sides (Remember $+ C$!!!)
  3. Solve for y (if possible)
  4. Use the initial condition to find C.

- Other notes:

Examples:

1. Solve the differential equation $\frac{dy}{dx} = -2y$ if $y(0) = 1$.

2. Solve the differential equation $\frac{dx}{dt} + x = 1$ if $x(1) = 0.1$.

3. Solve the differential equation $\frac{du}{dt} = u + ut^2$ if $u(0) = 5$.

4. Solve the differential equation $\frac{dy}{dx} = xe^y$ if $y(0) = 0$.

5. Solve the differential equation $\frac{ds}{d\theta} = -s^2 \tan \theta$ if $s(0) = 2$.

6. A tank contains 20 kg of salt dissolved in 5000 L of water. Brine that contains 0.03 kg of salt per liter of water enteres the tank at a rate of 25 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt remains in the tank after half an hour?

7. Find an equation of the curve that passes through the point $(0, 1)$ and whose slope at (x, y) is xy.