Math 2300 Spring 2018, Exam 2

March 12, 2018

PRINT YOUR NAME: ______________________________

PRINT INSTRUCTOR'S NAME: ______________________________

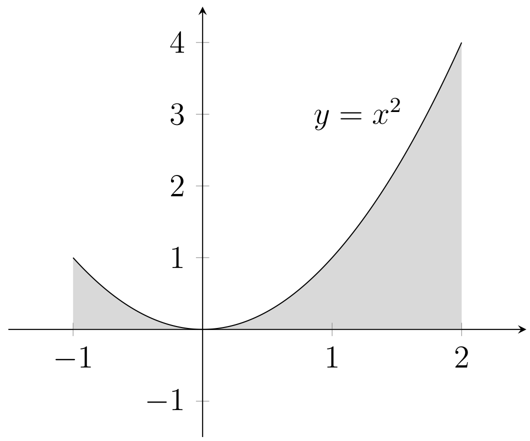
Mark your section/instructor:

☐	Section 001	Kevin Manley	8:00 - 8:50
☐	Section 002	Brendt Gerics	8:00 - 8:50
☐	Section 003	Faan Tone Liu	9:00 - 9:50
☐	Section 004	Shen Lu	9:00 - 9:50
☐	Section 005	Faan Tone Liu	10:00 - 10:50
☐	Section 006	Kevin Manley	10:00 - 10:50
☐	Section 007	Noah Williams	10:00 - 10:50
☐	Section 008	Ilia Mishev	11:00 - 11:50
☐	Section 009	Lee Roberson	11:00 - 11:50
☐	Section 010	Pedro Berrizbeita	12:00 - 12:50
☐	Section 011	Trubee Davison	12:00 - 12:50
☐	Section 012	Lee Roberson	1:00 - 1:50
☐	Section 013	Pedro Berrizbeita	1:00 - 1:50
☐	Section 014	Matthew Pierson	2:00 - 2:50
☐	Section 015	Isabel Corona	2:00 - 2:50
☐	Section 016	Robert Hines	3:00 - 3:50
☐	Section 017	Ruofan Li	3:00 - 3:50
☐	Section 018	Trevor Jack	4:00 - 4:50
☐	Section 019	Ilia Mishev	4:00 - 4:50
☐	Section 020	Jun Hong	4:00 - 4:50
☐	Section 430R	Patrick Newberry	10:00 - 10:50
☐	Section 800	Trubee Davison	9:00 - 9:50
☐	Section 888R	Ilia Mishev	2:00 - 2:50

Question	Points	Score
1	9	
2	8	
3	12	
4	8	
5	14	
6	12	
7	6	
8	4	
9	10	
10	7	
11	10	
Total:	100	

- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit. Use full mathematical or English sentences.
- You have 90 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions. For example leave fractions like 100/7 or expressions like ln(3)/2 as is.
- When done, give your exam to your instructor, who will mark your name off on a photo roster.
- We hope you show us your best work!

1. (9 points) Consider the region bounded by $y = 0$, $x = -1$, $x = 2$ and $y = x^2$. This region has been given below, and is assumed to have constant density, ρ.



(i) Find the x-coordinate of the center of mass, denoted by $\bar{x}$, for this region. You may use the fact that the area of this region is 3.

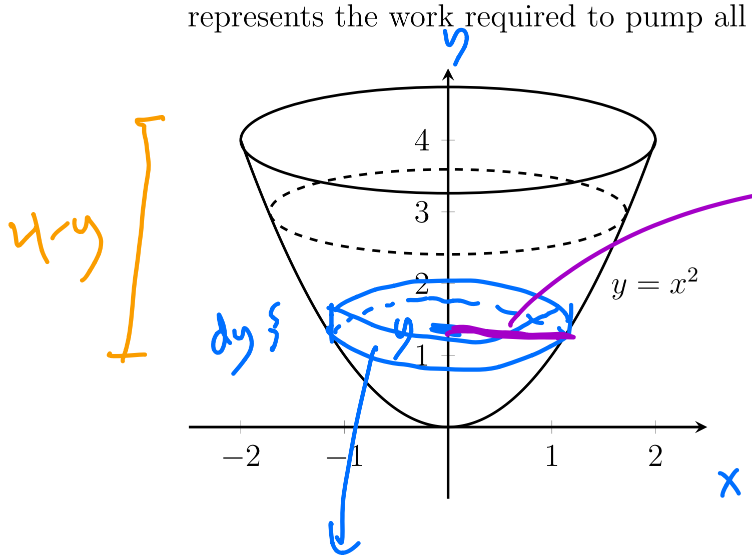
$$\bar{x} = \frac{1}{A}\int_{x=-1}^{x=2} x\,f(x)\,dx = \frac{1}{3}\int_{x=-1}^{x=2} x(x^2)\,dx$$

$$= \frac{1}{3}\left(\frac{x^4}{4}\Big]_{-1}^{2}\right) = \frac{5}{4}$$

(ii) **CIRCLE ONE.** Choose the interval which contains the y-coordinate of center of mass, denoted by $\bar{y}$, for this region.

(A) $[-2, 0)$
(B) $[0, 2)$
(C) $[2, 4)$
(D) $[4, 6)$
(E) None of the above

2. (8 points) Let R be the region bounded by $y = x^2$, $y = 4$, and $x = 0$. Rotate R around the y-axis to produce the paraboloid tank pictured below, which has a radius of 2 m and a height of 4 m. The tank is filled to a depth of 3 m with fluid of density $\rho\ \frac{kg}{m^3}$. Assume gravitational acceleration is $g\ \frac{m}{s^2}$. **Set up**, **but do NOT evaluate**, the integral that represents the work required to pump all the fluid out of the tank.



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$y = x^2$

$x = \sqrt{y}$

Volume = $\pi(\sqrt{y})^2 dy$

$$\int_{y=0}^{y=3} \rho g (4-y) \pi y \, dy = \text{Work}$$

3. (12 points) MULTIPLE CHOICE: For each sequence below, circle the correct answer. Note that these are **SEQUENCES**. They are **NOT** series.

(i) $a_n = \dfrac{\sin(n^2)}{\sqrt{n}}$

(A) Sequence converges to 0.

(B) Sequence converges, but not to 0.

(C) Sequence diverges to ∞

(D) Sequence diverges, but not to ∞.

(ii) $a_n = \dfrac{n}{\ln(n)}$

(A) Sequence converges to 0.

(B) Sequence converges, but not to 0.

(C) Sequence diverges to ∞

(D) Sequence diverges, but not to ∞.

(iii) $a_n = \dfrac{\sqrt{n^2+1}}{3n-1} \longrightarrow \dfrac{1}{3}$

(A) Sequence converges to 0.

(B) Sequence converges, but not to 0.

(C) Sequence diverges to ∞

(D) Sequence diverges, but not to ∞.

(iv) $a_n = \dfrac{(-1)^n n^2}{n^2+1}$

(A) Sequence converges to 0.

(B) Sequence converges, but not to 0.

(C) Sequence diverges to ∞

(D) Sequence diverges, but not to ∞.

4. (8 points) The following statements are both **FALSE**. Justify why each statement is false by providing an explanation. This explanation must include a specific example of a sequence or series.

   (i) If $\{a_n\}_{n=1}^{\infty}$ is a decreasing sequence, then it is convergent.

   (ii) If the sequence $\{a_n\}_{n=1}^{\infty}$ converges to zero, then the series $\sum_{n=1}^{\infty} a_n$ converges.

5. (14 points) Fill in the blanks to make the following sentences true:

(i) The series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3+n+1}}$ is ________________ (divergent/convergent),

by the Limit Comparison Test with $b_n =$ ________________.

(ii) The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n}$ is ________________ (divergent/convergent),

by the Direct Comparison Test with $b_n =$ ________________.

(iii) The series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^3}$ is ________________ (divergent/convergent),

because it is a ____________________________ (geometric series/p-series),

with (circle one)

(A) $p \leq 1$

(B) $p > 1$

(C) $|r| < 1$

(D) $|r| \geq 1$

6. (12 points) MULTIPLE CHOICE: Fill in the blank and circle ONE series test to make the sentence true:

(i) The series $\displaystyle\sum_{n=1}^{\infty} \frac{3^n \cdot n}{n!}$ is ________________ (divergent/convergent) by the

(A) Ratio Test
(B) Limit Comparison Test
(C) Test for Divergence
(D) Integral Test

(ii) The series $\displaystyle\sum_{n=2}^{\infty} (-1)^n \cos\left(\frac{\pi}{n}\right)$ is ______________________ (divergent/convergent) by the

(A) Ratio test
(B) Alternating Series Test
(C) Direct Comparison Test
(D) Test for Divergence

(iii) The series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^3}$ is ______________________ (divergent/convergent) by the

(A) Alternating Series Test
(B) Direct Comparison Test
(C) Test for Divergence
(D) Integral Test

7. (6 points) MULTIPLE CHOICE: Determine if the following series converge, or diverge. If they converge, determine what they converge to.

(i) $\sum_{n=1}^{\infty}(\arctan(n+1) - \arctan(n))$

(A) The series converges to 0.

(B) The series converges to $\frac{\pi}{4}$.

(C) The series converges to $\frac{\pi}{2}$.

(D) The series converges to π.

(E) The series diverges.

(ii) $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{2^{n-1}}$

(A) The series converges to 0.

(B) The series converges to -2.

(C) The series converges to $\dfrac{2}{5}$.

(D) The series converges to $-\dfrac{3}{5}$.

(E) The series diverges.

8. (4 points) Suppose the series $\sum_{n=1}^{\infty} a_n$ has partial sums $s_N = 7 + \dfrac{2}{\sqrt{N}}$. What does the series $\sum_{n=1}^{\infty} a_n$ converge to?

9. (10 points) Determine whether the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{5n^2 - 2n}{7n^3 + 4}$$

(You MUST verify the hypotheses of any test you use.)

10. (7 points) Use the following series to answer the problems below.

$$5 - \frac{5}{3} + \frac{5}{9} - \frac{5}{27} + \frac{5}{81} + \cdots$$

(a) Rewrite the series using sigma notation.

(b) Determine whether the series converges or diverges. If the series converges, find its sum. (You MUST verify the hypotheses of any test you use.)

11. (10 points) Consider the alternating series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{\sqrt[3]{n}}$.

(a) Show that this alternating series converges.

(b) Determine whether this series converges absolutely or conditionally.

(c) Use the Alternating Series Estimation Theorem to determine how large n must be, so that the estimation error $|R_n|$ is less than or equal to $\frac{1}{1000}$.