Math 2300, Midterm 2

March 11, 2019

PRINT YOUR NAME: ______________________________

PRINT INSTRUCTOR'S NAME: ______________________________

Mark your section/instructor:

☐	Section 001	Kevin Manley	8:00 - 8:50	☐	Section 014	Joel Ornstein	2:00 - 2:50
☐	Section 002	Jun Hong	8:00 - 8:50	☐	Section 015	Matthew Pierson	2:00 - 2:50
☐	Section 003	Albany Thompson	9:00 - 9:50	☐	Section 016	Sarah Arpin	2:00 - 2:50
☐	Section 004	Robert Hines	9:00 - 9:50	☐	Section 017	Corey Lyons	3:00 - 3:50
☐	Section 005	Michael Wheeler	9:00 - 9:50	☐	Section 018	Ilia Mishev	3:00 - 3:50
☐	Section 006	Joseph Timmer	11:00 - 11:50	☐	Section 019	Pedro Berrizbeitia	3:00 - 3:50
☐	Section 007	Andrew Campbell	11:00 - 11:50	☐	Section 020	André Davis	4:00 - 4:50
☐	Section 008	Braden Balentine	11:00 - 11:50	☐	Section 021	Leo Herr	4:00 - 4:50
☐	Section 009	Lucas Gagnon	12:00 - 12:50	☐	Section 022	Sangman Lee	4:00 - 4:50
☐	Section 010	Corey Lyons	12:00 - 12:50	☐	Section 430R	Patrick Newberry	10:00 - 10:50
☐	Section 011	Joseph Timmer	1:00 - 1:50	☐	Section 888R	Ilia Mishev	2:00 - 2:50
☐	Section 012	Jonathan Quartin	1:00 - 1:50	☐	Section 889R	Ilia Mishev	4:00 - 4:50
☐	Section 013	Pedro Berrizbeitia	1:00 - 1:50				

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	10	12	12	12	11	12	13	6	12	100
Score:										

- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit, except where otherwise indicated. Use full mathematical or English sentences.
- You have 90 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions unless otherwise stated. For example leave fractions like 100/7 or expressions like $\ln(3)/2$ as is.
- When done, give your exam to the proctor, who will mark your name off on a photo roster.

1. Let R be the region bounded by the x-axis, and the graph of $y = 1 - x^2$. Assume the density of the region is $\rho = 1$.

   (a) (4 points) What is the mass of the region R? Circle your response.

   (I) 1/3

   (II) 1

   (III) 2/3

   (IV) 4/3

   (b) (3 points) What is the x-coordinate of the center of mass of the region R? Circle your response.

   (I) 4/15

   (II) 0

   (III) 8/15

   (IV) 6/15

   (c) (3 points) What is the y-coordinate of the center of mass of the region R? Circle your response.

   (I) 4/15

   (II) 0

   (III) 8/15

   (IV) 6/15

2. (12 points) Determine if the series $\sum_{k=2}^{\infty} \frac{1}{k\left(\ln(k)\right)^2}$ is convergent or divergent. Show your work.

3. (12 points) Use the **Limit Comparison Test** to determine whether the series $\displaystyle\sum_{k=1}^{\infty} \frac{k^2+2k+1}{k^4-k+3}$ is convergent or divergent. Show your work.

4. For each sequence a_k below, circle the true statement. You do not need to show your work.

(a) (3 points) $a_k = \tan(k\pi)$

(I) The sequence converges to 0.

(II) The sequence converges, but not to 0.

(III) The sequence diverges to ∞.

(IV) The sequence diverges, but not to ∞.

(b) (3 points) $a_k = \dfrac{k}{\ln(k)}$

(I) The sequence converges to 0.

(II) The sequence converges, but not to 0.

(III) The sequence diverges to ∞.

(IV) The sequence diverges, but not to ∞.

(c) (3 points) $a_k = \dfrac{\sqrt{k^2+1}}{3k-1}$

(I) The sequence converges to 0.

(II) The sequence converges, but not to 0.

(III) The sequence diverges to ∞.

(IV) The sequence diverges, but not to ∞.

(d) (3 points) $a_k = \arctan k$

(I) The sequence converges to 0.

(II) The sequence converges, but not to 0.

(III) The sequence diverges to ∞.

(IV) The sequence diverges, but not to ∞.

5. Consider the series $\sum_{k=1}^{\infty} \frac{(-1)^k k^2}{4k^3 - 2}$.

(a) (8 points) Does the series converge absolutely, converge conditionally, or diverge? Justify your answer.

(b) (3 points) Using the Alternating Series Estimation Theorem, what is the maximal error in using $\sum_{k=1}^{4} \frac{(-1)^k k^2}{4k^3 - 2}$ to estimate the value of the series above? Circle your response.

(I) $\frac{9}{106}$

(II) $\frac{25}{498}$

(III) $\frac{16}{254}$

(IV) $\frac{36}{862}$

6. Fill in the blanks to make the following sentences true:

(a) (4 points) The series $\sum_{n=2}^{\infty}(-1)^n \cos\left(\frac{\pi}{n}\right)$ is __________________ (divergent/convergent)

by the (circle one)

(I) Ratio Test.

(II) Alternating Series Test.

(III) Direct Comparison Test.

(IV) Test for Divergence.

(b) (4 points) The series $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3+n+1}}$ is ______________ (divergent/convergent),

by the Limit Comparison Test with $b_n =$ ______________.

(c) (4 points) The series $\sum_{k=1}^{\infty} (-\cos(1))^k$ is ______________ (divergent/convergent),

because it is a _______________________ (geometric series/p-series),

with (circle one)

(I) $p \leq 1$.

(II) $p > 1$.

(III) $|r| < 1$.

(IV) $|r| \geq 1$.

7. (13 points) Show the series $\displaystyle\sum_{k=1}^{\infty} \frac{e^k}{k!}$ is convergent. Show your work.

8. (6 points) For the series $\sum_{k=1}^{\infty} a_k$ the n^{th} partial sum is given to be $s_n = 2 - \dfrac{1}{\sqrt{n}}$. Circle the true statement and fill in the blank where appropriate.

(I) The series $\sum_{k=1}^{\infty} a_k$ is convergent and its sum is ____________.

(II) The series $\sum_{k=1}^{\infty} a_k$ is convergent, but not enough information is given to determine its sum.

(III) The series $\sum_{k=1}^{\infty} a_k$ is divergent.

(IV) The series $\sum_{k=1}^{\infty} a_k$ is divergent and its sum is ____________.

(V) Not enough information is given to determine whether the series $\sum_{k=1}^{\infty} a_k$ converges.

9. (12 points) The tank depicted below is full of water. How much work is required to empty the tank if water is to be pumped out of the spout? You may assume that the mass density of water is 1000 kilograms per cubic meter and that the acceleration due to gravity is $g = 9.8\text{m/sec}^2$. (**Set up, but do not evaluate the integral.**)