Math 2300, Midterm 2

October 21, 2019

PRINT YOUR NAME: ______________________________

PRINT INSTRUCTOR'S NAME: ______________________________

Mark your section/instructor:

☐	Section 001	Jun Hong	8:00 - 8:50	☐	Section 009	Isabelle Kraus	11:00 - 11:50
☐	Section 002	Trubee Davison	8:00 - 8:50	☐	Section 010	Hunter Davenport	12:00 - 12:50
☐	Section 003	Osita Onyejekwe	8:00 - 8:50	☐	Section 011	Corey Lyons	1:00 - 1:50
☐	Section 004	Osita Onyejekwe	9:00 - 9:50	☐	Section 012	Pedro Berrizbeitia	1:00 - 1:50
☐	Section 005	Joseph Timmer	9:00 - 9:50	☐	Section 013	Sarah Arpin	2:00 - 2:50
☐	Section 006	Pedro Berrizbeitia	9:00 - 9:50	☐	Section 014	Alexander Nita	2:00 - 2:50
☐	Section 007	Pedro Berrizbeitia	11:00 - 11:50	☐	Section 015	Pedro Berrizbeitia	3:00 - 3:50
☐	Section 008	Trubee Davison	11:00 - 11:50	☐	Section 016	Sangman Lee	4:00 - 4:50

Question:	1	2	3	4	5	6	7	8	Total
Points:	12	12	12	12	13	13	13	13	100
Score:									

- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit, except where otherwise indicated. Use full mathematical or English sentences.
- You have 90 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions unless otherwise stated. For example leave fractions like 100/7 or expressions like ln(3)/2 as is.
- When done, give your exam to the proctor, who will mark your name off on a photo roster.

1. For each sequence a_k below, circle the true statement. You do not need to show your work.

   (a) (3 points) $a_k = \sin(k\pi)$

   (I) The sequence converges to 0.

   (II) The sequence converges, but not to 0.

   (III) The sequence diverges to ∞.

   (IV) The sequence diverges, but not to ∞.

   (b) (3 points) $a_k = \dfrac{k}{[\ln(k)]^2}$

   (I) The sequence converges to 0.

   (II) The sequence converges, but not to 0.

   (III) The sequence diverges to ∞.

   (IV) The sequence diverges, but not to ∞.

   (c) (3 points) $a_k = \dfrac{\sqrt[3]{k^3-4}}{2k+5}$

   (I) The sequence converges to 0.

   (II) The sequence converges, but not to 0.

   (III) The sequence diverges to ∞.

   (IV) The sequence diverges, but not to ∞.

   (d) (3 points) $a_k = \dfrac{1}{\arctan k}$

   (I) The sequence converges to 0.

   (II) The sequence converges, but not to 0.

   (III) The sequence diverges to ∞.

   (IV) The sequence diverges, but not to ∞.

2. Fill in the blanks to make the following sentences true. You do not need to show your work.

(a) (4 points) The series $\sum_{n=2}^{\infty}(-1)^n \sin^2\left(\frac{\pi}{2} - \frac{1}{n^3}\right)$ is ____________________ (divergent/convergent) by the (circle one)

(I) Ratio Test.

(II) Alternating Series Test.

(III) Direct Comparison Test.

(IV) Test for Divergence.

(V) Integral Test

(b) (4 points) The series $\sum_{n=1}^{\infty} \frac{n^2+n-1}{\sqrt{n^5+n-1}}$ is ________________ (divergent/convergent),

by the Limit Comparison Test with $b_n =$ ________________.

(c) (4 points) The series $\sum_{k=1}^{\infty} \frac{2^{4k}}{3^{2k-3}}$ is ________________ (divergent/convergent),

because it is a __________________________ (geometric series/p-series),

with (circle one)

(I) $p \leq 1$.

(II) $p > 1$.

(III) $|r| < 1$.

(IV) $|r| \geq 1$.

3. (12 points) Determine the behavior of the following series. You do not need to show your work.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$ converges conditionally / converges absolutely / diverges

$$\sum_{n=0}^{\infty} \frac{8^n}{n!}$$ converges conditionally / converges absolutely / diverges

$$\sum_{n=1}^{\infty} \frac{n^5}{5^n}$$ converges conditionally / converges absolutely / diverges

$$\sum_{n=1}^{\infty} \frac{1}{n}$$ converges conditionally / converges absolutely / diverges

$$\sum_{n=0}^{\infty} \frac{3n^2+n+1}{2n^{7/2}+n^{5/2}+1}$$ converges conditionally / converges absolutely / diverges

$$\sum_{k=1}^{\infty} k\left(\frac{-2}{3}\right)^k$$ converges conditionally / converges absolutely / diverges

4. Fill in the blanks to make the following sentences true. You do not need to show your work.

(a) (4 points) The series $\displaystyle\sum_{n=2}^{\infty} 4\left(\frac{2^n}{5^{n-1}}\right)$ is ____________________ (divergent/convergent).

The limit of the sequence of partial sums is (circle one).

(I) 0

(II) 3/2

(III) 16/3

(IV) 6

(V) 40/3

(VI) ∞

(b) (4 points) The series $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^2+2n}$ is ____________________ (divergent/convergent).

The limit of the sequence of partial sums is (circle one).

(I) 0

(II) 1/2

(III) 1

(IV) 3/2

(V) 6

(VI) ∞

(c) (4 points) The series $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right)$ is ____________________ (divergent/convergent).

The limit of the sequence of partial sums is (circle one).

(I) 0

(II) 1

(III) $\ln 2$

(IV) e

(V) $\ln 3$

(VI) ∞

5. Suppose that 2 J of work is needed to stretch a spring from its natural length of 0.30 m to a length of 0.42 m.

   Recall that a J has units $\dfrac{\mathrm{kg}\cdot\mathrm{m}^2}{s^2}$ and a N has units $\dfrac{\mathrm{kg}\cdot\mathrm{m}}{s^2}$.

   (a) (4 points) Find the constant k of the spring. Include units in your response. You do not need to simplify your answer.

   (b) (4 points) How much work is needed to stretch the spring from 0.35 m to 0.40 m? Include units in your response. You do not need to simplify your answer.

   (c) (5 points) How far beyond its natural length will a force of exactly 30 N keep the spring stretched? Include units in your response. You do not need to simplify your answer.

6. (13 points) Use the integral test to determine if the following series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{1}{n \ln(\sqrt{n})}$$

7. (13 points) Consider the series $\sum_{k=2}^{\infty}(-1)^k\frac{\ln k}{k}$. Does the series converge absolutely, converge conditionally, or diverge? Justify your answer.

8. (13 points) Determine if the following series converges or diverges. Justify your answer.

$$\sum_{n=1}^{\infty} \frac{2^n \cdot n!}{(2n)!}$$