Math 2300, Final Exam

May 8, 2017

PRINT YOUR NAME: ______________________________

PRINT INSTRUCTOR'S NAME: ______________________________

Mark your section/instructor:

☐	Section 001	Sarah Salmon	8:00 - 8:50
☐	Section 002	Shawn Burkett	9:00 - 9:50
☐	Section 003	Katharine Adamyk	10:00 - 10:50
☐	Section 004	Al Bronstein	11:00 - 11:50
☐	Section 005	Cherry Ng	11:00 - 11:50
☐	Section 006	Saeed Khalili	12:00 - 12:50
☐	Section 007	Al Bronstein	1:00 - 1:50
☐	Section 008	Mason Pelfrey	1:00 - 1:50
☐	Section 009	Trubee Davison	2:00 - 2:50
☐	Section 010	Ilia Mishev	3:00 - 3:50
☐	Section 011	Mark Pullins	4:00 - 4:50
☐	Section 012	John Willis	10:00 - 10:50
☐	Section 013	Ilia Mishev	12:00 - 12:50
☐	Section 014	Hanson Smith	2:00 - 2:50
☐	Section 015	Kevin Manley	3:00 - 3:50
☐	Section 016	Kevin Manley	10:00 - 10:50
☐	Section 018	Sebastian Bozlee	8:00 - 8:50
☐	Section 019	Joseph Timmer	4:00 - 4:50
☐	Section 800	Trubee Davison	9:00 - 9:50
☐	Section 430R	Patrick Newberry	10:00 - 10:50
☐	Section 888R	Ilia Mishev	2:00 - 2:50

Question	Points	Score
1	10	
2	6	
3	12	
4	8	
5	10	
6	8	
7	6	
8	6	
9	12	
10	6	
11	8	
12	8	
Total:	100	

- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit, except where otherwise indicated. Use full mathematical or English sentences.
- You have 150 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions unless otherwise stated. For example leave fractions like 100/8 or expressions like ln(4)/2 as is.
- When done, give your exam to your instructor, who will mark your name off on a photo roster.
- We hope you show us your best work!

1. Consider the region bounded by the graphs of $f(x) = x^2 + 1$ and $g(x) = 3 - x^2$.

   (a) (5 points) Write the integral for the volume of the solid of revolution obtained by rotating this region about the x-axis. Do not evaluate the integral.

   (b) (5 points) Write the integral for the volume of the solid of revolution obtained by rotating this region about the line $x = 3$. Do not evaluate the integral.

2. **MULTIPLE CHOICE:** Circle the best answer.

(a) (1 point) Is the following integral an improper integral?

$$\int_{-1}^{1} \frac{1}{x^2}\, dx$$

(A) Yes

(B) No

(b) (5 points) Evaluate the integral. $\displaystyle\int_{-1}^{1} \frac{1}{x^2}\, dx =$

3. Consider the curve parameterized by $\begin{cases} x = \frac{1}{3}t^3 + 3t^2 + \frac{2}{3} \\ y = t^3 - t^2 \end{cases}$ for $0 \le t \le \sqrt{5}$.

(a) (6 points) Find an equation for the line tangent to the curve when $t = 1$.

(b) (3 points) Compute $\dfrac{d^2y}{dx^2}$ at $t = 1$.

(c) (3 points) Write an integral to compute the total arc length of the curve. Do not evaluate the integral.

4. Consider the function $f(x) = x^2 \arctan(x)$.

   (a) (5 points) Find a power series representation for $f(x)$.

   (b) (3 points) What is $f^{(83)}(0)$, the 83^{rd} derivative of $f(x)$ at $x = 0$?

5. A tank contains 200 L of salt water with a concentration of 4 g/L. Salt water with a concentration of 3 g/L is being pumped into the tank at the rate of 8 L/min, and the tank is being emptied at the rate of 8 L/min. Assume the contents of the tank are being mixed thoroughly and continuously. Let $S(t)$ be the amount of salt (measured in grams) in the tank at time t (measured in minutes).

   (a) (1 point) What is the amount of salt in the tank at time $t = 0$?

   (b) (2 points) What is the rate at which salt enters the tank?

   (c) (2 points) What is the rate at which salt leaves the tank?

   (d) (1 point) What is $\frac{dS}{dt}$, the net change of salt in the tank?

   (e) (4 points) Write an initial value problem relating $S(t)$ and $\frac{dS}{dt}$. Solve the initial value problem.

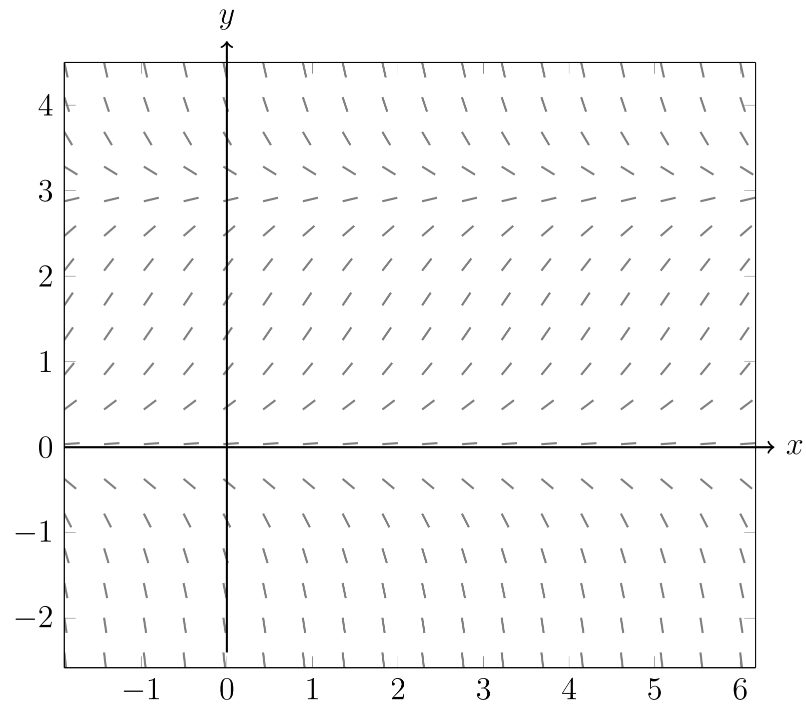
6. Compute the following integrals.

(a) (4 points) $\displaystyle\int \sin^3(x)\cos^2(x)\,dx$

(b) (4 points) $\displaystyle\int \frac{x+1}{x^2(x-1)}\,dx$

7. A slope field for the differential equation $y' = 2y\left(1 - \frac{y}{3}\right)$ is shown below.

$$y' = 2y\left(1 - \frac{y}{3}\right)$$



(a) (2 points) Sketch the graph of the solution that satisfies following initial condition. Label the solution as (a).

$$y(0) = 1$$

(b) (2 points) Sketch the graph of the solution that satisfies the following initial condition. Label the solution as (b).

$$y(2) = -1$$

(c) (2 points) Given the initial condition $y(0) = c$, for what values of c is $\lim_{x\to\infty} y(x)$ finite?

8. Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^4}$.

(a) (3 points) Use the Integral Test Remainder Estimate to find an upper bound for the error in using S_{10} (the 10^{th} partial sum) to approximate the sum of this series.

(b) (3 points) How many terms are required to ensure that the sum is accurate to within 0.1?

9. Determine whether the series is convergent or divergent and circle the corresponding answer. Then write the test allows one to determine convergence or divergence.

   (a) (3 points) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$

   (A) convergent (B) divergent Test: ____________________________

   (b) (3 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(n+1)}{n^2-3}$

   (A) convergent (B) divergent Test: ____________________________

   (c) (3 points) $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{5}{n}\right)$

   (A) convergent (B) divergent Test: ____________________________

   (d) (3 points) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+5}{(n+2)!}$

   (A) convergent (B) divergent Test: ____________________________

10. **MULTIPLE CHOICE:** Circle the best answer below.

(a) (2 points) The sequence $a_n = 1 - 0.2^n$

(A) converges to 0. (B) converges, but not to 0. (C) diverges.

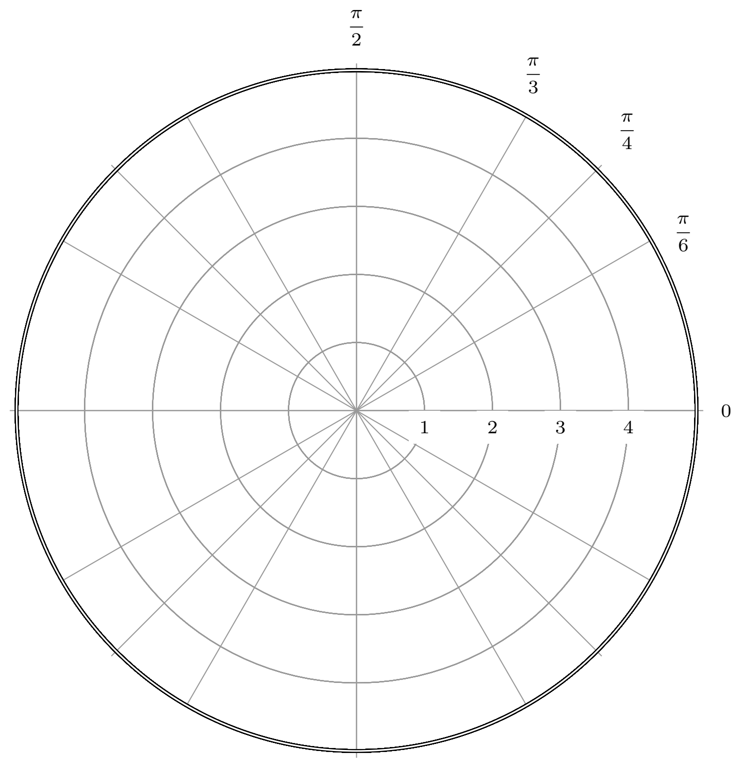
(b) (2 points) The sequence $a_n = \dfrac{3n-4}{2n-1}$

(A) converges to 0. (B) converges, but not to 0. (C) diverges.

(c) (2 points) The sequence $a_n = n + \dfrac{1}{n}$

(A) converges to 0. (B) converges, but not to 0. (C) diverges.

11. (a) (4 points) Sketch the curves $r = 2$ and $r = 3 + 2\sin\theta$ on the axes below.



(b) (4 points) Write an integral that represents the area contained outside the first curve ($r = 2$) and inside the second curve ($r = 3 + 2\sin(\theta)$). Do not evaluate the integral.

12. **MULTIPLE CHOICE:** Circle the best answer below.

(a) (2 points) Is the following statement ALWAYS, SOMETIMES, or NEVER true?

If $\sum |a_n|$ converges, then $\sum a_n$ also converges.

(A) ALWAYS (B) SOMETIMES (C) NEVER

(b) (2 points) Is the following statement ALWAYS, SOMETIMES, or NEVER true?

If $\sum a_n$ converges, then $\sum |a_n|$ also converges.

(A) ALWAYS (B) SOMETIMES (C) NEVER

(c) (2 points) The graph of $\begin{cases} x = t^2 - 3 \\ y = -t \end{cases}$ for $-\infty < t < \infty$ is a

(A) line (B) parabola (C) circle (D) ellipse

(d) (2 points) The graph of $\begin{cases} x = t^2 - 3 \\ y = -t^2 \end{cases}$ for $-\infty < t < \infty$ is a

(A) line (B) parabola (C) circle (D) ellipse