Math 2300, Midterm 3

April 17, 2017

PRINT YOUR NAME: ____________________

PRINT INSTRUCTOR'S NAME: ____________________

Mark your section/instructor:

☐	Section 001	Sarah Salmon	8:00 - 8:50
☐	Section 002	Shawn Burkett	9:00 - 9:50
☐	Section 003	Katharine Adamyk	10:00 - 10:50
☐	Section 004	Al Bronstein	11:00 - 11:50
☐	Section 005	Cherry Ng	11:00 - 11:50
☐	Section 006	Saeed Khalili	12:00 - 12:50
☐	Section 007	Al Bronstein	1:00 - 1:50
☐	Section 008	Mason Pelfrey	1:00 - 1:50
☐	Section 009	Trubee Davison	2:00 - 2:50
☐	Section 010	Ilia Mishev	3:00 - 3:50
☐	Section 011	Mark Pullins	4:00 - 4:50
☐	Section 012	John Willis	10:00 - 10:50
☐	Section 013	Ilia Mishev	12:00 - 12:50
☐	Section 014	Hanson Smith	2:00 - 2:50
☐	Section 015	Kevin Manley	3:00 - 3:50
☐	Section 016	Kevin Manley	10:00 - 10:50
☐	Section 018	Sebastian Bozlee	8:00 - 8:50
☐	Section 019	Joseph Timmer	4:00 - 4:50
☐	Section 800	Trubee Davison	9:00 - 9:50
☐	Section 430R	Patrick Newberry	10:00 - 10:50
☐	Section 888R	Ilia Mishev	2:00 - 2:50

Question	Points	Score
1	12	
2	6	
3	8	
4	7	
5	7	
6	12	
7	9	
8	11	
9	12	
10	6	
11	10	
Total:	100	

- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit, except where otherwise indicated. Use full mathematical or English sentences.
- You have 90 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions unless otherwise stated. For example leave fractions like 100/8 or expressions like $\ln(4)/2$ as is.
- When done, give your exam to your instructor, who will mark your name off on a photo roster.
- We hope you show us your best work!

1. (12 points) Find the interval of convergence for the following power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{3^n\sqrt{n+1}}(x-1)^n$$

2. Circle the power series representation for the following functions $f(x)$, centered at 0.

(a) (3 points) $f(x) = xe^x$

(A) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$ (B) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n-1}x^n}{n!}$

(C) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$ (D) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)!}$

(b) (3 points) $f(x) = \dfrac{1}{1+x^2}$

(A) $\displaystyle\sum_{n=0}^{\infty} (-1)^{2n}x^{2n}$ (B) $\displaystyle\sum_{n=0}^{\infty} (-1)^{n}x^{2n}$

(C) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$ (D) $\displaystyle\sum_{n=0}^{\infty} (-1)^{2n}x^{n}$

3. (8 points) Find the sum of the series

$$\sum_{n=1}^{\infty} n \left(\frac{1}{3}\right)^{n-1}$$

4. Let $f(x) = \sin(2x)$.

(a) (4 points) Write the Taylor series of $f(x) = \sin(2x)$ centered at 0 using sigma notation.

(b) (3 points) What is $f^{(45)}(0)$ (the 45^{th} derivative of f at 0)?

5. (7 points) Find the coefficient of x^3 in the Taylor series of $f(x) = \dfrac{x}{9+x}$ centered at 0.

6. Suppose the power series $\sum b_n(x-2)^n$ converges when $x=-4$ and diverges when $x=10$.

(a) (2 points) The series $\sum b_n$

(A) Converges

(B) Diverges

(C) Cannot determine convergence or divergence

(b) (2 points) At $x=-8$, the power series $\sum b_n(x-2)^n$

(A) Converges

(B) Diverges

(C) Cannot determine convergence or divergence

(c) (2 points) At $x=8$, the power series $\sum b_n(x-2)^n$

(A) Converges

(B) Diverges

(C) Cannot determine convergence or divergence

(d) (3 points) What is the **largest** possible radius of convergence of the power series $\sum b_n(x-2)^n$?

(A) 1

(B) 2

(C) 6

(D) 8

(E) 10

(F) 14

(e) (3 points) What is the **smallest** possible radius of convergence of the power series $\sum b_n(x-2)^n$?

(A) 0

(B) 1

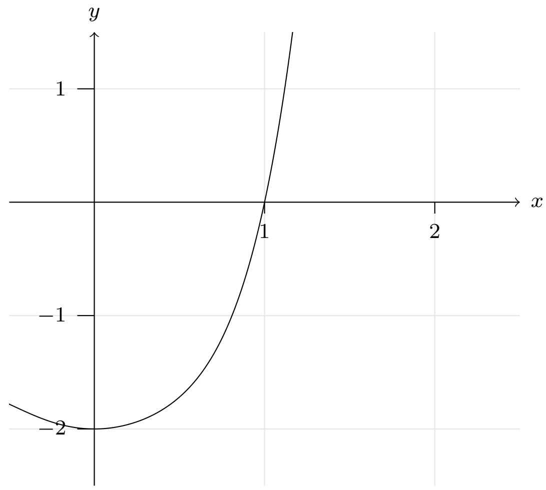
(C) 2

(D) 4

(E) 6

(F) 8

7. Let $T_3(x) = a + bx + cx^2 + dx^3$ be the 3rd degree Taylor polynomial for $f(x)$ centered at 0. If the curve below is $f(x)$, then answer the following questions:



(a) (3 points) What is the value for a? Explain your answer.

(b) (3 points) What is the value for b? Explain your answer.

(c) (3 points) What is a possible value for c? Explain your answer.

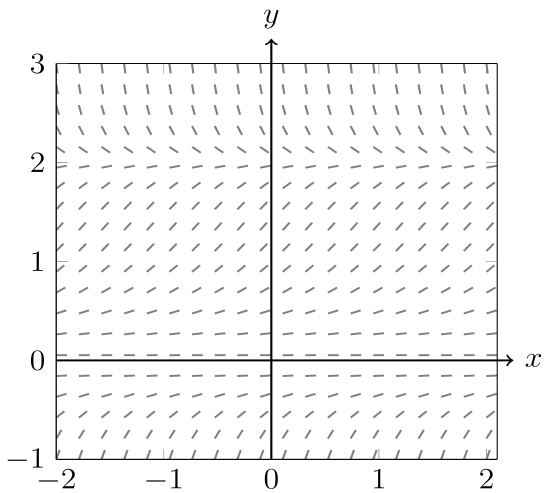
8. Let $f(x) = \sin(x)$.

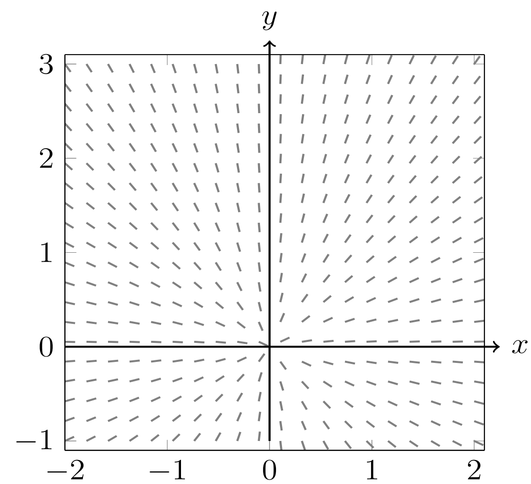
(a) (2 points) Find an upper bound M for $|f^{(n+1)}(x)|$ on $[0, \frac{1}{2}]$.

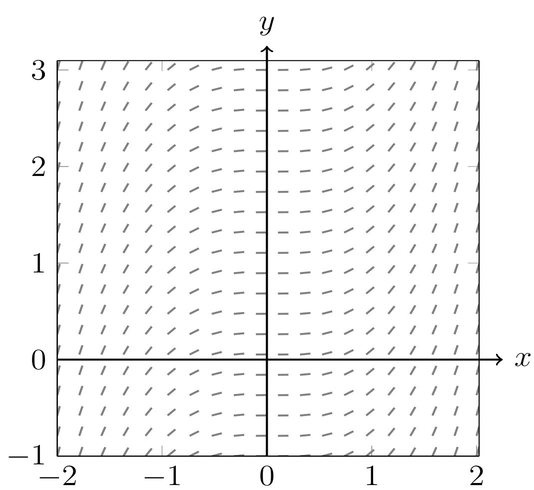
(b) (7 points) Let $T_n(x)$ denote the n^{th} degree Taylor polynomial for f centered at 0 and let $R_n(x) = f(x) - T_n(x)$. Use Taylor's inequality to find an upper bound for $|R_n(\frac{1}{2})|$.

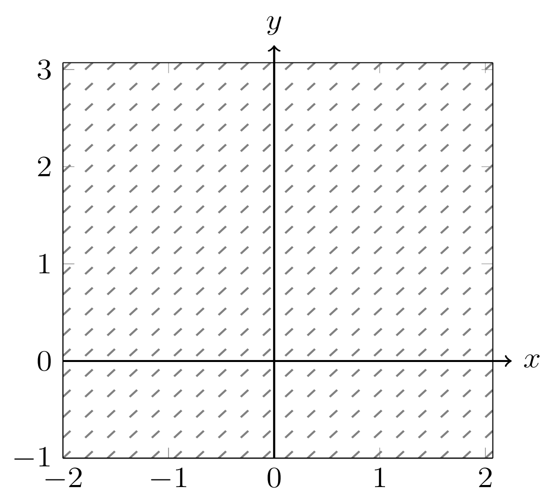
(c) (2 points) Does your answer to (b) allow you to determine whether or not $f(x)$ agrees with its Taylor series at $x = \frac{1}{2}$? Why or why not?

9. (12 points) For each of the following slope fields, give the letter of its corresponding differential equation.









(A) $y' = 1$ (B) $y' = x$ (C) $y' = x^2$

(D) $y' = y/x$ (E) $y' = y^2(2 - y)$

10. Circle the solution to the following differential equations or initial value problems.

(a) (3 points) $y' = 5y, \qquad y(0) = -2$

(A) $y = 5e^{2x}$ (B) $y = 5e^{-2x}$ (C) $y = -5e^{2x}$

(D) $y = 2e^{5x}$ (E) $y = 2e^{-5x}$ (F) $y = -2e^{5x}$

(b) (3 points) $y' = -\dfrac{x^3}{y^5}$

(A) $y^6/6 = C - x^4/4$ (B) $y^5/5 = C - x^3/3$ (C) $y^4/4 = C - x^4/4$

(D) $y^6 = \dfrac{1}{C - x^2}$ (E) $y^5 = \dfrac{1}{C - x^3}$ (F) $y^4 = \dfrac{1}{C - x^4}$

11. (a) (8 points) Solve the following initial value problem for y

$$y' = \frac{3x^2 + 4x - 4}{2y}, \qquad y(1) = -2.$$

(b) (2 points) Find $y(2)$.