1. **(10 pts) Determine if the following statements are always true, sometimes true, or never true.**

   (a) If $\int_1^\infty f(x)\,dx$ diverges and $f(n) = a_n$, then $\sum_{n=1}^{\infty} a_n$ converges.

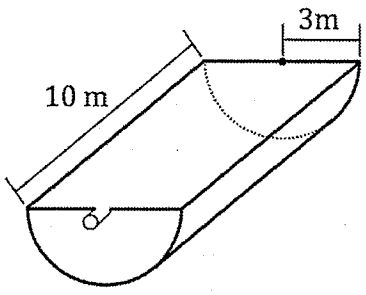
   (b) If $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

   (c) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} |a_n|$ converges.

   (d) If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$, then $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge or both diverge.

   (e) $\sum_{n=1}^{\infty} (a_n - a_{n+1}) = a_1$

3. (12 points) A 10m long cylindrical tank with semi-circular cross sections with radius 3m (as shown in the diagram) is lying on the ground. Assume that the spout adds no height to the tank. The density of water is $\rho = 1000$ kg/m^3, and acceleration due to gravity is $g = 9.8$ m/s^2. If the tank is full of water, set up an integral which represents the work required to pump the water out of the tank. **You do not need to evaluate the integral.**



4. (12 points) For each of the sequences below, choose the answer that correctly describes the sequence. You do not need to show your work.

(a) $\left\{\dfrac{2\sin(n)}{\sqrt[3]{n}}\right\}_{n=1}^{\infty}$

i. Converges to 0

ii. Converges, but not to 0

iii. Diverges to ∞

iv. Diverges, but not to ∞

(b) $\{(-2)^n\}_{n=1}^{\infty}$

i. Converges to 0

ii. Converges, but not to 0

iii. Diverges to ∞

iv. Diverges, but not to ∞

(c) $\left\{\dfrac{n^n}{n!}\right\}_{n=1}^{\infty}$

i. Converges to 0

ii. Converges, but not to 0

iii. Diverges to ∞

iv. Diverges, but not to ∞

(d) $\left\{\dfrac{3n^2-5n+2}{-4n^2+10n-2}\right\}_{n=1}^{\infty}$

i. Converges to 0

ii. Converges, but not to 0

iii. Diverges to ∞

iv. Diverges, but not to ∞

5. (18 points) Determine if the following series are convergent or divergent. **If the series is convergent, find its sum.**

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(n-1)}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{3^{n+2}}{2^{2n-1}}$

(c) $\sum_{n=2}^{\infty} \frac{n^2+3}{n(n-1)}$

6. (5 points) Determine the upper bound, provided by the Alternating Series Estimation Theorem, for the magnitude of the error in using S_{19} (the 19th partial sum) to approximate the alternating series $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n\ln(n+1)}$. Circle the correct answer.

(a) $(-1)^{19}\frac{1}{19\ln(20)}$

(b) $\frac{1}{19\ln(20)}$

(c) $\frac{1}{20\ln(21)}$

(d) $-\frac{1}{20\ln(21)}$

(e) $\int_{19}^{\infty} \frac{1}{x\ln(x+1)}\,dx$

(f) None of the above

7. (14 points) Determine whether the following series are absolutely convergent, convergent but not absolutely convergent, or divergent. **In your answer, please specify which test you are using.**

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 5^n n!}{(2n)!}$

8. (18 points) Determine if the following series are convergent or divergent. **In your answer, please specify which test you are using.**

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+3n+1}{\sqrt{n^6+2n+1}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan(n)}{n^2}$

9. (5 points) Find the y-coordinate of the centroid, $\bar{y}$, of the region bounded by $y = \sqrt{1-x^2}$ and the x-axis, for $0 \leq x \leq 1$. Circle the correct answer.

(a) $\frac{1}{3}$

(b) $\frac{\pi}{4}$

(c) $\frac{3}{4\pi}$

(d) $\frac{3\pi}{4}$

(e) $\frac{4}{3\pi}$

(f) None of the above

10. (6 points) If $\lim_{n\to\infty} a_n = 0$, must $\sum_{n=1}^{\infty} a_n$ converge? If so, provide a reason why. If not, give an example of a series where $\lim_{n\to\infty} a_n = 0$, but $\sum_{n=1}^{\infty} a_n$ diverges.