MATH 2300 – review problems for Exam 2

1. A metal plate of constant density ρ (in gm/cm^2) has a shape bounded by the curve $y = \sqrt{x}$, the x-axis, and the line $x = 1$.

   (a) Find the mass of the plate. Include units.

   $$\text{Mass} = \text{Density} \times \text{Area} = \rho \times \int_0^1 \sqrt{x}\, dx = \frac{2}{3}\rho \text{ gm}$$

   (b) Find the center of mass of the plate. Include units.

   $$\overline{x} = \frac{\rho \int_0^1 x\sqrt{x}\, dx}{\frac{2}{3}\rho} = \frac{3}{5} \text{ cm}, \qquad \overline{y} = \frac{\rho \int_0^1 \frac{1}{2}(\sqrt{x})^2\, dx}{\frac{2}{3}\rho} = \frac{3}{8} \text{ cm}$$

2. Write down (but do not evaluate) a definite integral representing the arc length of the function $f(x) = \ln x$ between $x = 1$ and $x = 5$.

   $$\int_1^5 \sqrt{1 + (f'(x))^2}\, dx = \int_1^5 \sqrt{1 + \left(\frac{1}{x}\right)^2}\, dx$$

3. If $\displaystyle\int_1^5 f(x)\, dx = 7$ and $\displaystyle\int_5^4 f(x)\, dx = 5$, then find the average value of $f(x)$ on $[1, 4]$.

   $f_{ave} = \frac{1}{4-1}\int_1^4 f(x)\, dx = \frac{1}{3}(\int_1^5 f(x)\, dx - \int_4^5 f(x)\, dx) = \frac{1}{3}(\int_1^5 f(x)\, dx + \int_5^4 f(x)\, dx) = \frac{1}{3}(7 + 5) = 4.$

4. Find the average value of $f(x) = 3^{-x}$ on the interval $[1, 3]$. $f_{ave} = \frac{1}{3-1}\int_1^3 3^{-x}\, dx = \frac{1}{2}\left[-\frac{3^{-x}}{\ln(3)}\right]_1^3 = \frac{1}{2}\left(\frac{1}{3\ln(3)} - \frac{1}{27\ln(3)}\right) \approx .135.$

5. **(Exercise 7 from Section 6.6 in Stewart's Calculus Concepts and Contexts)** Suppose that 2J of work is needed to stretch a spring from its natural length of 30 cm to a length of 42 cm.

   (a) How much work is needed to stretch the spring from 35 cm to 40 cm?
   (b) How far beyond its natural length will a force of 30 N keep the spring stretched?

6. **(Exercise 11 from Section 6.6 in Stewart's Calculus Concepts and Contexts)** Show how to approximate the required work by a Riemann sum. Then express the work as an integral and evaluate it: a heavy rope, 50 ft long, weighs 0.5 lb/ft and hangs over the edge of a building 120 ft high.

   (a) How much work is done in pulling the rope to the top of the building?
   (b) How much work is done in pulling half the rope to the top of the building?

7. **(Exercise 15 from Section 6.6 in Stewart's Calculus Concepts and Contexts)** Show how to approximate the required work by a Riemann sum. Then express the work as an integral and evaluate it: a leaky 10-kg bucket is lifted from the ground to a height of 12m at a constant speed with a rope that weighs 0.8 kg/m. Initially the bucket contains 36 kg of water, but the water leads at a constant rate and finishes draining just as the bucket reaches the 12-m level. How much work is done?

8. Complete Exercise 19 from Section 6.6 on pg. 473 in Stewart's Calculus Concepts and Contexts

   Find the answers to exercises 5-8 (and all odd-numbered exercises in the textbook) in the back of the textbook.

9. Find the limit of all the sequences in the sequence activity:
   http://math.colorado.edu/math2300/projects/SequencesPractice.pdf

10. Does $\{a_n\}$, where $a_n = \frac{1}{n}$, converge? If so, what does it converge to? I won't be fooled - a_n is a sequence, not a series. I am just being asked if the sequence has a limit. Yes, it converges to 0. $\lim_{n\to\infty} \frac{1}{n} = 0$. If I had been asked about the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n}$, then that answer would have been divergent, since it is the harmonic series.

11. Decide whether each of the following sequences converges. If a series converges, what does it converge to? If not, why not?

    (a) The sequence whose n-th term is $a_n = 1 - \frac{1}{n}$. Converges to 1. (The $\frac{1}{n}$ part goes to zero.)

    (b) The sequence whose n-th term is $b_n = \sqrt{n+1} - \sqrt{n}$. Converges to 0:

    $$\sqrt{n+1} - \sqrt{n} = (\sqrt{n+1} - \sqrt{n}) \left(\frac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+1} + \sqrt{n}} \right) = \frac{n+1-n}{\sqrt{n+1} + \sqrt{n}} = \frac{1}{\sqrt{n+1} + \sqrt{n}},$$

    and the denominator of this expression grows as n gets large, so it approaches zero.

    (c) The sequence whose n-th term is $c_n = \cos(\pi n)$. Diverges. The sequence is $(-1, 1, -1, 1, \dots)$ which oscillates.

    (d) The sequence $\{d_n\}$, where $d_1 = 2$ and

    $$d_n = 2d_{n-1} \quad \text{for } n > 1.$$

    Diverges. The sequence is $(2, 4, 8, 16, \dots)$, a geometric sequence with $r > 1$, so the terms go to infinity.

12. Find the sum of the series. For what values of the variable does the series converge to this sum?

    (a) $1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} + \cdots$

    (a) $\frac{2}{2-x}$, for $|x| < 2$

    (b) $y - y^2 + y^3 - y^4 + \cdots$

    (b) $\frac{y}{y+1}$, for $|y| < 1$

    (c) $4 + z + \frac{z^2}{3} + \frac{z^3}{9} + \cdots$

    (c) $\frac{z-12}{z-3}$, for $|z| < 3$

13. For each of the following series, determine whether or not they converge. If they converge, determine what they converge to.

   (a) $\sum_{n=1}^{\infty} 5\left(\frac{2}{3}\right)^{n-1}$ This is a geometric series with first term $a = 5$ and ratio $r = \frac{2}{3}$. $|r| < 1$, so by the geometric series test the series converges to $\frac{a}{1-r} = \frac{5}{1-\frac{2}{3}} = 15$.

   (b) $\sum_{n=2}^{\infty} 3\frac{4^{n+1}}{5^{n-4}}$ This is a geometric series. To get the first term I substitute $n = 2$ to get $a = 3 \cdot 4^3/5^{-2} = 3 \cdot 64 \cdot 25 = 4800$. The ratio is $r = \frac{4}{5}$. $|r| < 1$ so this series converges by the geometric series test to $\frac{a}{1-r} = \frac{4800}{1-\frac{4}{5}} = 24000$.

   (c) $\sum_{n=3}^{\infty} \frac{7(-\pi)^{2n-1}}{e^{3n+1}} = \sum_{n=3}^{\infty} \frac{7(-\pi)^{2n}}{-\pi \cdot e \cdot e^{3n}} = \sum_{n=3}^{\infty} \frac{7((-\pi)^2)^n}{-\pi \cdot e \cdot (e^3)^n} \sum_{n=3}^{\infty} \frac{7(\pi^2)^n}{-\pi \cdot e \cdot (e^3)^n}$. Written in this form it is clear that this is a geometric series. Substitute $n = 3$ to find that the first term is $a = \frac{-7\pi^5}{e^7}$. $r = \frac{\pi^2}{e^3} < 1$, so by the Geometric Series Test, this series converges to $\frac{a}{1-r} = \frac{-7\pi^5}{e^7} \cdot \frac{1}{1-\frac{\pi^2}{e^3}}$

   (d) $\sum_{n=2}^{8} 4(.07)^{n+1}$ This is a finite geometric series with the first term $a = 4(.07)^3$ and $r = .07$. The sum is $\frac{\text{first term}-\text{first unadded term}}{1-r} = \frac{4(.07)^3-4(.07)^{10}}{1-.07}$.

   (e) $\sum_{n=1}^{\infty} \frac{1}{n^2+3n+2}$. Using partial fractions decomposition, the series equals $\sum_{n=1}^{\infty}\left(\frac{1}{n+1} - \frac{1}{n+2}\right)$. Calculating partial sums for this telescoping series: $s_1 = \frac{1}{2} - \frac{1}{3}$, $s_2 = \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} = \frac{1}{2} - \frac{1}{4}$. More generally $s_n = \frac{1}{2} - \frac{1}{n+2}$. Taking limits, $\sum_{n=1}^{\infty} \frac{1}{n^2+3n+2} = \lim_{n\to\infty}\left(\frac{1}{2} - \frac{1}{n+2}\right) = \frac{1}{2}$

14. For each of the following series, determine if it converges absolutely, converges conditionally, or diverges. Completely justify your answers, including all details.

   (a) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n+5}$ The series does not converge absolutely, as $\sum_{n=2}^{\infty}\left|\frac{(-1)^n}{n+5}\right| = \sum_{n=2}^{\infty}\frac{1}{n+5}$ diverges by the Limit Comparison Test with $\sum_{n=2}^{\infty}\frac{1}{n}$ (which diverges by the p-test). However, it converges by the Alternating Series Test, since $b_{n+1} = \frac{1}{n+6} \le \frac{1}{n+5} = b_n$ for all n and $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{n+5} = 0$ Thus, the series converges conditionally.

   (b) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ The function $f(x) = \frac{1}{x\ln(x)^2}$ is continuous, positive, and decreasing on $[2, \infty)$, and the integral $\int_2^{\infty} \frac{1}{x\ln(x)^2}dx$ converges to $\frac{1}{\ln(2)}$. Thus, the series converges by the Integral Test. Since all terms in the series are positive, this implies that the series absolutely converges.

   (c) $\sum_{n=2}^{\infty} \frac{n}{(\ln n)^2}$ Using L'Hôpitals Rule, we have that $\lim_{n\to\infty} \frac{n}{\ln(n)^2} = \lim_{n\to\infty} \frac{1}{2\ln(n)\cdot\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{2\ln(n)} = \lim_{n\to\infty} \frac{1}{2\cdot\frac{1}{n}} = \lim_{n\to\infty} \frac{1}{2}n = \infty$. Thus, this series diverges by the Test for Divergence.

(d) $\sum_{n=1}^{\infty} \frac{2n^2(-3)^n}{n!}$ We use the Ratio Test: $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{2(n+1)^2 3^{n+1}}{(n+1)!} \cdot \frac{n!}{2n^2 \cdot 3^n} = \lim_{n\to\infty} \frac{3(n+1)^2}{(n+1)n^2} = \lim_{n\to\infty} \frac{3(n+1)}{n^2} = 0 < 1$. Thus, the series converges absolutely by the Ratio Test.

(e) $\sum_{n=1}^{\infty} \left(\frac{4 \cdot 2^n}{(-3)^{n+1}} + \frac{1}{2^n} \right)$ We can write this series as the sum of the series $\sum_{n=1}^{\infty} \frac{4 \cdot 2^n}{(-3)^{n+1}}$ and $\sum_{n=1}^{\infty} \frac{1}{2^n}$. The latter series is a geometric series with $a = r = \frac{1}{2}$, thus it converges, therefore converges absolutely (all of its terms are positive). The former series converges absolutely as well, since $\sum_{n=1}^{\infty} \left| \frac{4 \cdot 2^n}{(-3)^{n+1}} \right| = \sum_{n=1}^{\infty} \frac{4 \cdot 2^n}{3^{n+1}} = \sum_{n=1}^{\infty} \frac{4 \cdot 2}{3^2} \left(\frac{2}{3}\right)^{n-1} = \sum_{n=1}^{\infty} \frac{8}{9} \left(\frac{2}{3}\right)^{n-1}$, which converges since it is a geometric series with $r = \frac{2}{3} < 1$. Since the sum of two absolutely convergent series is absolutely convergent, our original series is absolutely convergent.

(f) $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n^3 + 2n}$ This series converges absolutely, as $\sum_{n=1}^{\infty} \left| \frac{(-1)^n \sqrt{n}}{n^3+2n} \right| = \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^3+2n}$, and this series converges via the Limit Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n^{\frac{5}{2}}}$ (which converges by the p-test).

(g) $\sum_{n=1}^{\infty} \frac{n + 3n^5}{2n^7 + 3}$ This series converges via the Limit Comparison Test with the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$, which we know converges by the p-test. Since this series has positive terms, it therefore converges absolutely.

(h) $\sum_{n=1}^{\infty} n^{\frac{1}{n}}$ Using L'Hôpital's Rule, we have that $\lim_{n\to\infty} \ln\left(n^{\frac{1}{n}}\right) = \lim_{n\to\infty} \frac{\ln(n)}{n} = \lim_{n\to\infty} \frac{\frac{1}{n}}{1} = 0$. Thus, $\lim_{n\to\infty} n^{\frac{1}{n}} = \lim_{n\to\infty} e^{\ln\left(n^{\frac{1}{n}}\right)} = e^{\lim_{n\to\infty} \ln\left(n^{\frac{1}{n}}\right)} = e^0 = 1 \neq 0$ so the series diverges by the Test for Divergence.

(i) $\sum_{n=1}^{\infty} \arctan n$ $\lim_{n\to\infty} \arctan n = \frac{\pi}{2} \neq 0$, thus the series diverges by the Test for Divergence.

(j) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ This series converges absolutely, as $0 \leq \sum_{n=1}^{\infty} \left| \frac{\sin(n)}{n^2} \right| = \sum_{n=1}^{\infty} \frac{|\sin(n)|}{n^2} \leq \sum_{n=1}^{\infty} \frac{1}{n^2}$, which converges by the p-test. Thus, $\sum_{n=1}^{\infty} \frac{|\sin(n)|}{n^2}$ converges by the Term-Size Comparison Test, therefore $\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$ converges absolutely.

(k) $\sum_{n=1}^{\infty} \left(\frac{n}{n+1} \right)^n$ $\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e \neq 0$, thus the series diverges by the Test for Divergence.

(l) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ We use the Ratio Test: $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{n!} = \lim_{n\to\infty} \frac{(n+1)}{(n+1)^{n+1}} \cdot n^n = \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e > 1$. Thus, the series diverges by the Ratio Test.

15. Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

(a) Confirm using the Alternate Series Test that the series converges. This is an alternating series with $b_n = \frac{1}{n!}$. Thus, since $b_{n+1} = \frac{1}{(n+1)!} \leq \frac{1}{n!} = b_n$ and $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{n!} = 0$, we have that the series converges by the Alternating Series Test.

(b) How many terms must be added to estimate the sum to within .0001? By the Alternating Series Estimation Theorem, the number of terms we need to add to estimate the sum within .0001 is the smallest integer n such that $b_{n+1} = \frac{1}{(n+1)!} < .0001$. Equivalently, we need to find the smallest n such that $(n+1)! > 10,000$. We note that $7! = 5,040$ and $8! = 40,320$, thus our desired n is $n = 7$. We must add 7 terms to estimate the sum to within .0001.

(c) Estimate the sum to within .0001. $S \approx S_7 = -\frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} - \frac{1}{7!} \approx -.63214$.

16. How many terms of $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ should be added to estimate the sum to within .01? No calculators.

By the Alternating Series Estimation Theorem, the number of terms we need to add to estimate the sum within .0001 is the smallest number n such that $b_{n+1} = \frac{1}{\sqrt{n+1}} < .01$. Equivalently, we need to find the smallest n such that $\sqrt{n+1} > 100$ or $n > 9,999$. Thus, we must add $10,000$ terms of of the series to estimate its sum within .01.

17. Check whether the following series converge or diverge. In each case, give the answer for convergence, and name the test you would use. If you use a comparison test, name the series $\sum b_n$ you would compare to.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n^2+1)}$ diverges – use integral or limit comparison test, comparing to $\frac{1}{n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^2}$ converges – term-size comparison test with $\frac{1}{n^3}$

(c) $\displaystyle\sum_{n=1}^{\infty} \left(n + \frac{1}{n}\right)^n$ diverges – divergence test (nth term test) (that is, the n the term does not go to zero)

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^4+1}}{5n^2}$ diverges – limit comparison test or nth term test

(e) $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$ (hint: consider $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$) converges – limit comparison test

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$ converges – ratio test

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{(n+3)!}$ diverges – ratio test

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(n+2)!}$ converges – simplify to $\frac{1}{(n+1)(n+2)}$ and then use term-size comparison test or limit comparison test.

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$ converges – ratio test

18. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$. Are the following statements true or false? Fully justify your answer.

(a) The series converges by limit comparison with the series $\sum_{n=1}^{\infty} \frac{1}{n}$. False

(b) The series converges by the ratio test. False

(c) The series converges by the integral test. False

19. Consider the series $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n}$. Are the following statements true or false? Fully justify your answer.

(a) The series converges by limit comparison with the series $\sum_{n=1}^{\infty} \frac{1}{n}$.False

(b) The series converges by the ratio test.False

(c) The series converges by the integral test. False

(d) The series converges by the alternating series test.True

(e) The series converges absolutely. False

20. Suppose the series $\sum a_n$ is absolutely convergent. Are the following true or false? Explain.

(a) $\sum a_n$ is convergent.true, because if a series converges absolutely, it must converge

(b) The sequence a_n is convergent. true, because $\sum a_n$ converges, so by the Divergence Test, $\lim_{n\to\infty} = 0$.

(c) $\sum (-1)^n a_n$ is convergent.True, in fact it converges absolutely.

(d) The sequence a_n converges to 1. False, the sequence converges to 0 by the Divergence Test.

(e) $\sum a_n$ is conditionally convergent. False

(f) $\sum \frac{a_n}{n}$ converges. True, this can be shown using the term-size comparison test.

21. Does the following series converge or diverge?

$$\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$$

You must justify your answer to receive credit. Yes, by the ratio test, because, if a_n denotes the nth term of this series, then the limit of $|a_{n+1}/a_n|$ as $n \to \infty$ is zero (which is less than 1).

22. A ball is dropped from a height of 10 feet and bounces. Assume that there is no air resistance. Each bounce is $\frac{3}{4}$ of the height of the bounce before.

(1) Find an expression for the height to which the ball rises after it hits the floor for the nth time.

$H(n) = 10(\frac{3}{4})^n$

(2) Find an expression for the total vertical distance the ball has traveled when it hits the floor for the nth time.

$D(n) = 10 + (2 \cdot 10 \cdot (3/4))\frac{1-(3/4)^{n-1}}{1-(3/4)}$

(3) Using without proof the fact that a ball dropped from a height of h feet reaches the ground in $\sqrt{h}/4$ seconds: Will the ball bounce forever? If not, how long it will take for the ball to come to rest?

The ball will not bounce forever. The total time it bounces is given by

$$(\sqrt{10}/4) + a/(1-r)$$

with $a = (1/2)\sqrt{10(3/4)}$ and $r = \sqrt{3/4}$.

Want more practice? Here's some more!

23. In theory, drugs that decay exponentially always leave a residue in the body. However in practice, once the drug has been in the body for 5 half-lives, it is regarded as being eliminated. If a patient takes a tablet of the same drug every 5 half-lives forever, what is the upper limit to the amount of drug that can be in the body?

    Let P_n represent the percentage of the drug in the body after the nth tablet. Then

$$P_1 = 1(100 \text{ percent})$$
$$P_2 = 1(.5 * .5 * .5 * .5 * .5) + 1 = 1.03125$$
$$P_3 = 1.03125(.03125) + 1 = 1.03223$$

    So,

$$P_n = \frac{1 - (.03125)^n}{1 - .03125}.$$

    As $n \to \infty$, $P_n \to 1.0323$, so this is the maximum amount of the drug in the body.

24. Let $\{f_n\}$ be the sequence defined recursively by $f_1 = 5$ and $f_n = f_{n-1} + 2n + 4$.

    (a) Check that the sequence g_n whose n-th term is $g_n = n^2 + 3n + 1$ satisfies this recurrence relation, and that $g_1 = 5$. (This tells us $g_n = f_n$ for all n.)
    We check:

$$g_{n-1} + 2n + 4 = ((n-1)^2 + 5(n-1) - 1) + 2n + 4 = n^2 + 5n - 1 = g_n$$

    and plainly $g_1 = 5$

    (b) Use the result of part (a) to find f_{20} quickly.

$$f_{20} = 20^2 + 5 \cdot 20 - 1 = 499$$

25. Find the values of a for which the series converges/diverges:

    (a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{2a}\right)^n$

    (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{a}\left(\frac{1}{2}\right)^n$

    (c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2}{n}\right)^a$

    (d) $\displaystyle\sum_{n=1}^{\infty} (\ln a)^n$

    (e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(\ln n)^a}$

(f) $\sum_{n=1}^{\infty}(1+a^n)$

(g) $\sum_{n=1}^{\infty}(1+a)^n$

(h) $\sum_{n=1}^{\infty}n^{\ln a}$

(i) $\sum_{n=1}^{\infty}a^{\ln n}$

(a) $|a| > 1/2$

(b) $a \neq 0$

(c) $a > 1$

(d) $e^{-1} < a < e$

(e) $a > 1$

(f) diverges for all a

(g) $-2 < a < 0$

(h) $0 < a < e^{-1}$

(i) $0 < a < e^{-1}$

26. Using the table below, estimate the length of the curve given by $y = f(x)$ from $(3, 4)$ to $(6, 0.7)$.

x	3	3.5	4	4.5	5	5.5	6
$f(x)$	4	3.6	2.4	-1	-0.5	0	0.7
$f'(x)$	-0.8	-2.4	-6.8	1	1	1.4	-0.4

The appropriate arc length formula is

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

So, we can find a good estimate of the total length by using the table

$$\begin{aligned} L \approx &\sqrt{1+(-0.8)^2}(.5) + \sqrt{1+(-2.4)^2}(.5) + \sqrt{1+(-6.8)^2}(.5) \\ &+ \sqrt{1+(1)^2}(.5) + \sqrt{1+(1)^2}(.5) + \sqrt{1+(1.4)^2}(.5) \end{aligned}$$

27. Determine if these sequences converge absolutely, converge conditionally or diverge.

(a) $\sum_{n=1}^{\infty}\frac{\cos n}{n^2}$ converges absolutely, take the absolute value and compare to $\frac{1}{n^2}$ using the term-size comparison test

(b) $\sum_{n=1}^{\infty}(-1)^n \frac{n}{n^2+1}$ converges conditionally. Take the absolute value, then use limit comparison, comparing to $\frac{1}{n}$ to show it does not converge absolutely. Use alternating series test to show original series converges (and thus converges conditionally)

28. A steady wind blows a kite due east. The kite's height above ground from horizontal position $x = 0$ to $x = 80$ feet is given by

$$y = 150 - \frac{1}{40}(x - 50)^2$$

Find the distance traveled by the kite. Just set up the integral - don't evaluate.

This distance can be calculated by finding the arc length of the above curve from $x = 0$ to $x = 80$. Using the arc length formula we find the distance traveled to be

$$L = \int_0^{80} \sqrt{1 + (-\frac{1}{20}(x - 50))^2}\, dx$$

For each of the following statements, determine if it is true Always, Sometimes or Never.

29. If a sequence a_n converges, then the sequence $(-1)^n a_n$ also converges. Sometimes. Try $a_n = 1$ and $a_n = 0$.

30. If a sequence $(-1)^n a_n$ converges to 0, then the sequence a_n also converges to 0. Always

31. The average value of a function is negative. Sometimes. Try $f(x) = 1$ on $[0, 1]$, and $f(x) = -1$ on $[0, 1]$.

32. The geometric series $\sum_{n=1}^{\infty} 5r^{n-1}$ converges to $\frac{5}{1-r}$ Sometimes - true if $|r| < 1$, false if $|r| \geq 1$.

33. The geometric series $\sum_{n=1}^{\infty} \frac{c}{5^{n-1}}$ converges to $\frac{5c}{4}$ Always, by the geometric series test.

34. If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} |a_n|$ converges. Sometimes. Try $a_n = (-1)^n/n^2$, and $a_n = (-1)^n/n$.

35. If a series $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$ Always, by the divergence test.

36. If the series $\sum_{n=1}^{\infty}(-1)^n a_n$ is conditionally convergent, then the sequence a_n converges. Always, The series converges and so by the divergence test, $\{(-1)^n a_n\}$ must converge to 0, and so $\{a_n\}$ also converges to 0.

37. If the series $\sum_{n=1}^{\infty}(-1)^n a_n$ is convergent, then the series $\sum_{n=1}^{\infty} a_n$ is convergent. Sometimes. Try $\sum(-1)^n/n$ and $\sum(-1)^n/n^2$.

38. If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$, then $\sum_{n=1}^{\infty} a_n$ converges conditionally. Sometimes. Try $a_n = (-1)^n/n$ (cond. conv.) and $a_n = (-1)^n/n^2$ (abs. conv.) and $a_n = (-1)^n \cdot n$ (div.)

39. If the series $\sum_{n=1}^{\infty} |a_n|$ diverges, then the series $\sum_{n=1}^{\infty} a_n$ diverges. Sometimes. Try $\sum(-1)^n/n$ and $\sum 1/n$