

MATH 2300 – review problems for Exam 1, addendum

Here's one more review problem:

Recalling that the error bound for the Trapezoidal rule in approximating a definite integral is given by $|E_T| \leq \frac{K(b-a)^3}{12n^2}$, how large should n be to guarantee that a trapezoidal rule approximation of $\int_1^4 \frac{6x^{5/2}}{5} dx$ is accurate to at least within .01?

Recall K is a number such that

$$|f''(x)| \leq K \quad \text{on } [1,4]$$

$$f(x) = \frac{6}{5} x^{5/2}$$

$$f'(x) = 3x^{3/2}$$

$$f''(x) = \frac{9}{2} x^{1/2}$$

$$|f''(x)| = \frac{9}{2} \sqrt{x} \quad \frac{9}{2} \sqrt{x} \text{ is increasing on } [1,4]$$

so the max occurs at $x=4$.

$$|f''(x)| \leq \frac{9}{2} \sqrt{4} = 9 \quad \text{on } [1,4]. \quad K=9.$$

$$|E_T| \leq \frac{9(4-1)^3}{12n^2} = \frac{3 \cdot 3^3}{4n^2} = \frac{81}{4n^2}$$

$$\text{It suffices to get } \frac{81}{4n^2} \leq .01 = \frac{1}{100}$$

solving:

$$\frac{81 \cdot 100}{4} \leq n^2$$

$$n \geq \sqrt{\frac{81 \cdot 100}{4}} = 45.$$