MATH 2300: Calculus II, Fall 2014
MIDTERM #3

Wednesday, November 12, 2014

YOUR NAME:

Circle Your CORRECT Section

◯ **001** M. Pelfrey .... (9am)	◯ **006** S. Weinell .... (3pm)
◯ **002** E. Angel .... (10am)	◯ **007** C. Blakestad .... (8am)
◯ **003** E. Angel .... (11am)	◯ **008** P. Washabaugh .... (1pm)
◯ **004** J. Harper .... (12pm)	◯ **009** J. Harper .... (3pm)
◯ **005** B. Chhay .... (2pm)	◯ **010** K. Parker .... (4pm)

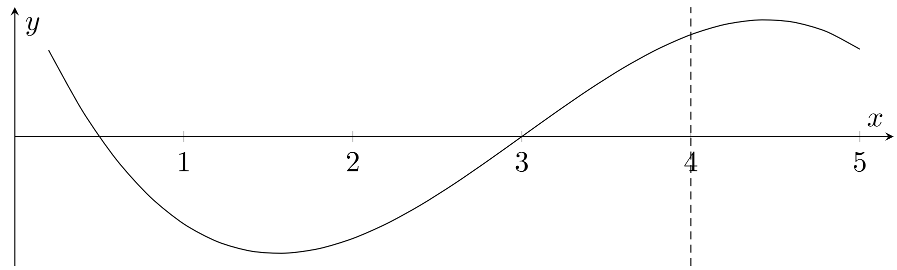
Important note: SHOW ALL WORK. BOX YOUR ANSWERS. Calculators are not allowed. No books, notes, etc. Throughout this exam, please provide exact answers where possible. That is: if the answer is 1/2, do not write 0.499 or something of that sort; if the answer is π, do not write 3.14159.

Problem	Points	Score
1	12	
2	8	
3	18	
4	10	
5	6	
6	12	
7	12	
8	12	
9	10	
TOTAL	100	

"On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work."

SIGNATURE:

1. (4 points each) Consider the function $y = f(x)$ sketched below.



Suppose $f(x)$ has Taylor series

$$f(x) = a_0 + a_1(x-4) + a_2(x-4)^2 + a_3(x-4)^3 + \cdots$$

about $x = 4$.

(a) Is a_0 positive or negative? Explain.

(b) Is a_1 positive or negative? Explain.

(c) Is a_2 positive or negative? Explain.

2. (8 points) The power series $\sum_{n=0}^{\infty} c_n(x+7)^n$ converges when at $x = -1$ and diverges at $x = -16$. Determine the maximal and minimal possible interval of convergences.

3. (3 points each) Do the following series converge or diverge? **CIRCLE** your answer, no work necessary.

a) $\sum_{n=1}^{\infty} \frac{1}{n}$ CONVERGE DIVERGE

b) $\sum_{n=0}^{\infty} \frac{5}{4^n}$ CONVERGE DIVERGE

c) $\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{\sqrt{n}}$ CONVERGE DIVERGE

d) $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$ CONVERGE DIVERGE

e) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ CONVERGE DIVERGE

f) $\sum_{n=1}^{\infty} \frac{10^n}{n!}$ CONVERGE DIVERGE

4. (10 points) Determine if the following series converges absolutely, conditionally, or diverges. **Justify your answer**.

$$\sum_{n=3}^{\infty} \frac{(-1)^n e^n}{(n-3)!}$$

5. (6 points) Which of the following series could be the Taylor series of $f(t) = t\cos(2t^2)$ about $x = 0$?

(A) $t - \frac{2^2 t^3}{2!} + \frac{2^4 t^5}{4!} - \frac{2^6 t^7}{6!} + \cdots$

(B) $t - \frac{2^2 t^5}{2!} + \frac{2^4 t^9}{4!} - \frac{2^6 t^{13}}{6!} + \cdots$

(C) $2t^2 - \frac{2^3 t^4}{3!} + \frac{2^5 t^6}{5!} - \frac{2^7 t^8}{7!} + \cdots$

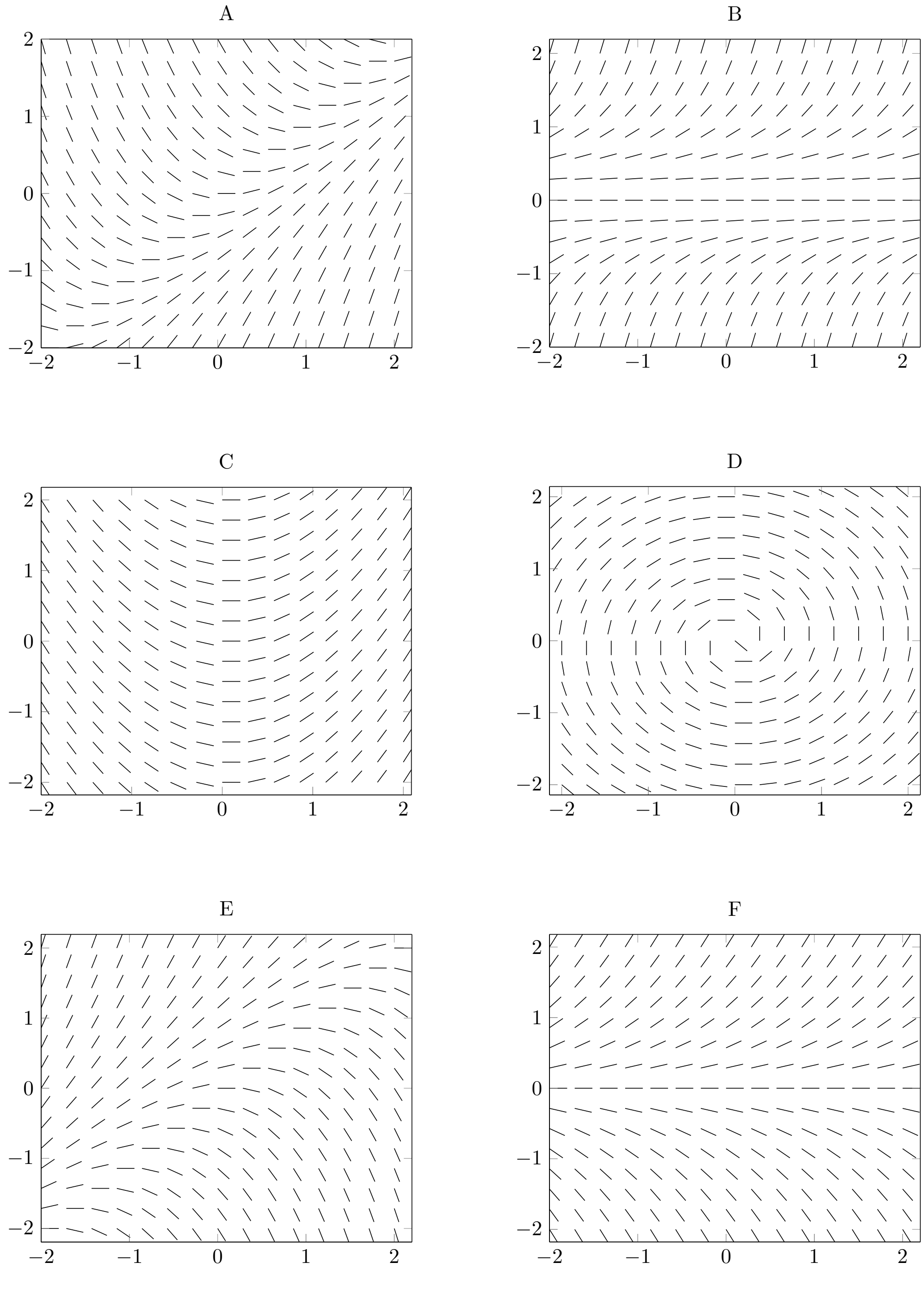
(D) $2t^2 - \frac{2^3 t^6}{3!} + \frac{2^5 t^{10}}{5!} - \frac{2^7 t^{14}}{7!} + \cdots$

(E) $t + 2t^2 + \frac{2^2 t^3}{2!} + \frac{2^3 t^4}{3!} + \cdots$

(F) $t + 2t^3 + \frac{2^2 t^5}{2!} + \frac{2^3 t^7}{3!} + \cdots$

6. (12 points) Use the integral test to determine if the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ converges or diverges.

7. (2 points each) Match the slope fields pictured to the differential equations.



$$\frac{dy}{dx} = -\frac{x}{y} \quad \_\_\_\_\_\_ \qquad \frac{dy}{dx} = x \quad \_\_\_\_\_\_ \qquad \frac{dy}{dx} = y^2 \quad \_\_\_\_\_\_$$

$$\frac{dy}{dx} = y \quad \_\_\_\_\_\_ \qquad \frac{dy}{dx} = x - y \quad \_\_\_\_\_\_ \qquad \frac{dy}{dx} = y - x \quad \_\_\_\_\_\_$$

8. (a) (5 points) Estimate $e^{0.1}$ using a third degree Taylor polynomial about 0.

(b) (7 points) Use the Lagrange error bound to find a bound on your approximation from part (a).

9. (2 points each) Each of the series in part (a)-(e) converges to a value in the list below. Choose the correct value for each series from the list and write your choice in the blank.

(a)

$$1 + \frac{\pi}{1} + \frac{\pi^2}{2} + \frac{\pi^3}{3!} + \frac{\pi^4}{4!} + \cdots + \frac{\pi^n}{n!} + \cdots = \underline{\hspace{3cm}}$$

(b)

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{(-1)^{n+1}}{n} + \cdots = \underline{\hspace{3cm}}$$

(c)

$$1 - \frac{\pi^2}{2} + \frac{\pi^4}{4!} - \frac{\pi^6}{6!} + \cdots + (-1)^n \frac{\pi^{2n}}{(2n)!} + \cdots = \underline{\hspace{3cm}}$$

(d)

$$10 - \frac{10^3}{3!} + \frac{10^5}{5!} - \frac{10^7}{7!} + \cdots + (-1)^n \frac{10^{2n+1}}{(2n+1)!} + \cdots = \underline{\hspace{3cm}}$$

(e)

$$1 - \frac{\pi}{1} + \frac{\pi^2}{2} - \frac{\pi^3}{3!} + \frac{\pi^4}{4!} - \cdots + (-1)^n \frac{\pi^n}{n!} + \cdots = \underline{\hspace{3cm}}$$

- e^{π}
- 0
- 1
- $\ln(1 + \pi)$
- -1
- $e^{-\pi}$

- $\ln(2)$
- $\sin(10)$
- $\cos(10)$
- e^{10}
- $\sin(1)$
- $\cos(1)$

- $\frac{1}{1 - \pi}$
- $\pi/4$
- $\frac{1}{1 - 10}$
- $-\pi/4$