MATH 2300: Calculus II, Fall 2014
MIDTERM #2

Wednesday, October 15, 2014

YOUR NAME:	Answer

Choose Your CORRECT Section

◯ **001** M. PELFREY .... (9AM)	◯ **006** S. WEINELL .... (3PM)
◯ **002** E. ANGEL .... (10AM)	◯ **007** C. BLAKESTAD .... (8AM)
◯ **003** E. ANGEL .... (11AM)	◯ **008** P. WASHABAUGH .... (1PM)
◯ **004** J. HARPER .... (12PM)	◯ **009** J. HARPER .... (3PM)
◯ **005** B. CHHAY .... (2PM)	◯ **010** K. PARKER .... (4PM)

Important note: SHOW ALL WORK. BOX YOUR ANSWERS. Calculators are not allowed. No books, notes, etc. Throughout this exam, please provide exact answers where possible. That is: if the answer is $1/2$, do not write 0.499 or something of that sort; if the answer is π, do not write 3.14159.

Problem	Points	Score
1	10	
2	14	
3	12	
4	11	
5	15	
6	10	
7	18	
8	10	
TOTAL	100	

"On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work."

SIGNATURE:

1. (2 points each) Match the recursively defined sequences in (a)-(e) to the sequences described in (I)-(V):

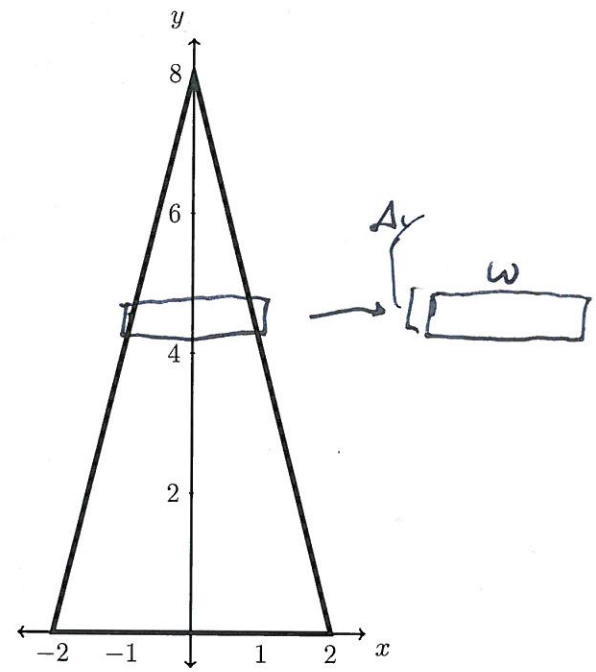
(a) $s_n = s_{n-1} + n$ for $n > 1$ and $s_1 = 1$	(I) $1, 3, 7, 15, \ldots$
(b) $s_n = s_{n-1} + \frac{1}{2}s_{n-2}$ for $n > 2$, $s_1 = 2$, $s_2 = 4$	(II) $2, 4, 7, 11, \ldots$
(c) $s_n = 2s_{n-1} + 1$ for $n > 1$ and $s_1 = 1$	(III) $1, 3, 6, 10, \ldots$
(d) $s_n = (s_{n-1})^2 - n$ for $n > 2$, $s_1 = 1$, $s_2 = 3$	(IV) $1, 3, 6, 32, \ldots$
(e) $s_n = s_{n-1} + n$ for $n > 1$ and $s_1 = 2$	(V) $2, 4, 5, 7, \ldots$

Write your answers below:

(a) III (b) V (c) I (d) IV (e) II

2. (14 points) Set up definite integrals computing the center of mass of an isosceles triangle whose density is δ g/cm^2, height is 8 cm, and base 4 cm. You do not need to solve the definite integrals.

$$\boxed{\text{By symmetry, } \bar{x} = 0\text{cm}}$$

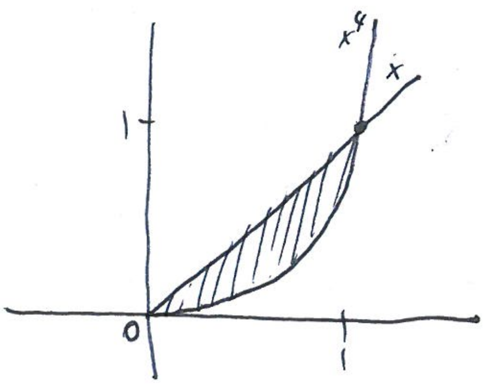


By similar triangles,

$$\frac{w}{8-y} = \frac{4}{8} \rightarrow w = \frac{1}{2}(8-y)$$

$$\boxed{\bar{y} = \frac{\int_0^8 \delta y w \, dy}{\int_0^8 \delta w \, dy} = \frac{\int_0^8 y\delta\left(\frac{1}{2}(8-y)\right) dy}{\int_0^8 \delta\left(\frac{1}{2}(8-y)\right) dy} \text{ cm}}$$

3. (12 points) The region bounded by the curves $y = x^4$ and $y = x$ is rotated around the x-axis. Compute the volume of the solid generated.



$$\text{outer volume} = \int_0^1 \pi (x)^2 dx$$

$$= \pi \frac{x^3}{3}\Big|_0^1 = \frac{\pi}{3}$$

$$\text{inner volume} = \int_0^1 \pi (x^4)^2 dx$$

$$= \pi \frac{x^9}{9}\Big|_0^1 = \frac{\pi}{9}$$

$$\text{outer} - \text{inner} = \boxed{\frac{2\pi}{9}}$$

4. (11 points total)

(a) Find the Taylor polynomial of degree 2 approximating $f(x) = \frac{2}{x+1}$ about $x = 0$.

$f'(x) = \frac{-2}{(x+1)^2}$ $\quad\quad f''(x) = \frac{4}{(x+1)^3}$

$P_2(x) = f(0) + f'(0)\cdot x + f''(0)\cdot\frac{x^2}{2}$

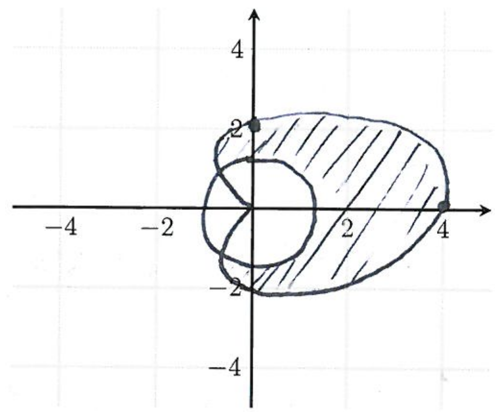
$\boxed{P_2(x) = 2 - 2x + 2x^3}$

(b) Find the Taylor polynomial of degree 2 approximating $f(x) = \frac{2}{x+1}$ about $x = 2$.

$q_2(x) = f(2) + f'(2)(x-2) + f''(2)\frac{(x-2)^2}{2}$

$\boxed{q_2(x) = \frac{2}{3} - \frac{2}{9}(x-2) + \frac{2}{27}(x-2)^2}$

5. (a) (3 points) Make a sketch of the cardioid $r = 2 + 2\cos\theta$ and the circle $r = 1$ on the coordinate plane below. Shade the area inside the cardioid $r = 2 + 2\cos\theta$ but outside the circle $r = 1$.



(b) (4 points) Calculate the polar coordinates of any points of intersection between $r = 2 + 2\cos\theta$ and $r = 1$.

For what θ does $2 + 2\cos\theta = 1$ hold?

$\rightarrow 2\cos\theta = -1$

$\rightarrow \cos\theta = -\frac{1}{2} \rightarrow \boxed{\theta = \frac{2\pi}{3}, \frac{4\pi}{3}}$

(c) (8 points) Set up an integral to compute the area inside $r = 2 + 2\cos\theta$ but outside $r = 1$. You do not need to solve the integral.

$$\int_0^{2\pi/3} \frac{1}{2}(2+2\cos\theta)^2\, d\theta - \int_0^{2\pi/3} \frac{1}{2}\, d\theta$$

$$+ \int_{4\pi/3}^{2\pi} \frac{1}{2}(2+2\cos\theta)^2\, d\theta - \int_{4\pi/3}^{2\pi} \frac{1}{2}\, d\theta$$

V

6. (a) (5 points) Simplify $-9+3-1+\frac{1}{3}-\cdots+\frac{1}{3^7}$. Write the best option:

(I) $9\left(\frac{1-(-1/3)^8}{1-(-1/3)}\right)$ (II) $-9\left(\frac{1-(-1/3)^8}{1-(-1/3)}\right)$ (III) $9\left(\frac{1-(-1/3)^9}{1-(-1/3)}\right)$

(IV) $-9\left(\frac{1-(-1/3)^9}{1-(-1/3)}\right)$ (V) $-9\left(\frac{1-(-1/3)^{10}}{1-(-1/3)}\right)$ (VI) $9\left(\frac{1-(-1/3)^{10}}{1-(-1/3)}\right)$

$$-9+3-1+\frac{1}{3}-\cdots+\frac{1}{3^7}$$

$$= -9\left(1-\frac{1}{3}+\frac{1}{9}-\frac{1}{27}+\cdots-\frac{1}{9\cdot 3^7}\right)$$

$$= -9\left(1-\frac{1}{3}+\frac{1}{3^2}-\frac{1}{3^3}+\cdots-\frac{1}{3^9}\right)$$

$$= \frac{-9\left(1-\left(-\frac{1}{3}\right)^{9+1}\right)}{1-\left(-\frac{1}{3}\right)} = \frac{-9\left(1-(-1/3)^{10}\right)}{1-(-1/3)}$$

(b) (5 points) Find the sum of the geometric series $8-2+\frac{1}{2}-\frac{1}{8}+\frac{1}{32}+\cdots$

$$= 8\left(1-\frac{1}{4}+\frac{1}{16}-\frac{1}{64}+\frac{1}{256}-\cdots\right)$$

$$= 8\left(1-\frac{1}{4}+\frac{1}{4^2}-\frac{1}{4^3}+\frac{1}{4^4}-\cdots\right)$$

$$= \frac{8}{1-\left(-\frac{1}{4}\right)} = \frac{8}{\left(\frac{5}{4}\right)} = \boxed{\frac{32}{5}}$$

7. (6 points each) The following integrals represent the volume of either a sphere or a cone and the variable of integration measures length. In each case, **CIRCLE** which shape is represented and give the radius of the sphere or the radius and height of the cone. Make a sketch to support your answer showing the variable and all other relevant quantities.

(a) $\int_0^1 \pi(2x)^2 dx$

- SPHERE

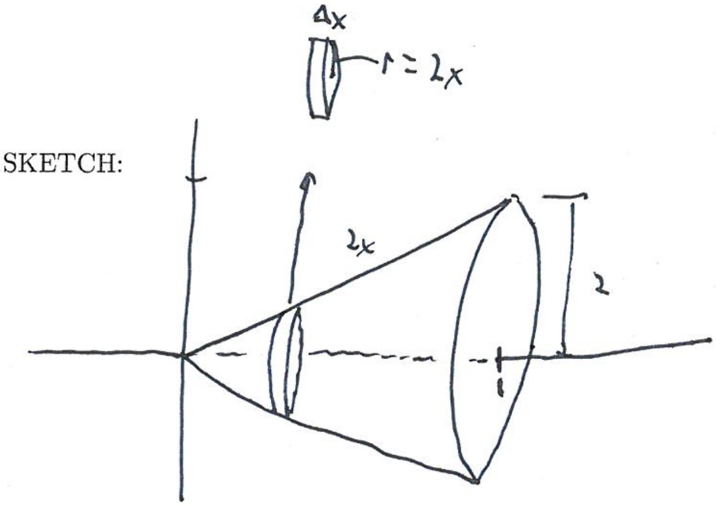
  Radius =

- CONE

  Radius = 2

  Height = 1

SKETCH:



(b) $\int_{-4}^{4} \pi(16 - h^2) dh$

- SPHERE

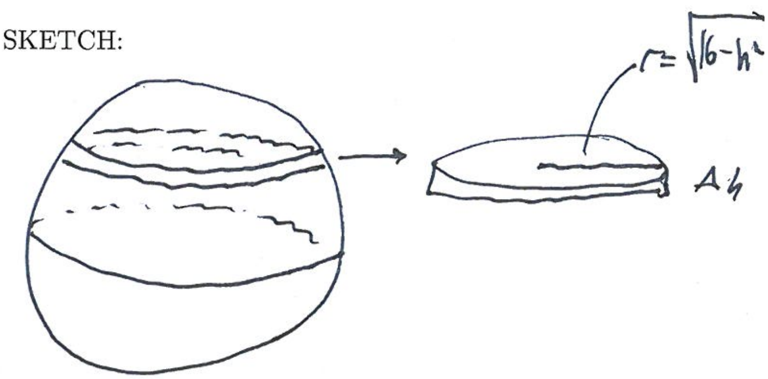
  Radius = 4

- CONE

  Radius =

  Height =

SKETCH:



(c) $\int_0^5 \pi \left(\frac{5-y}{5}\right)^2 dy$

- SPHERE

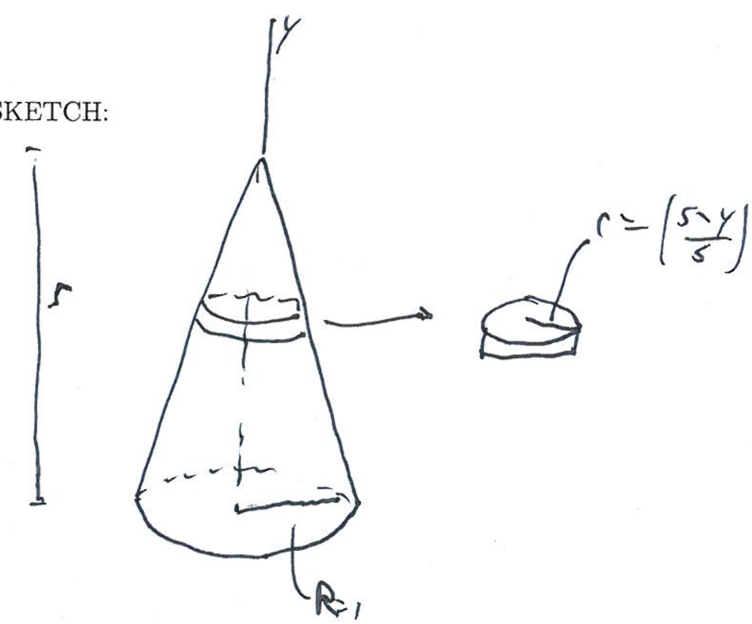
  Radius =

- CONE

  Radius = 1

  Height = 5

SKETCH:



8. (10 points total) No justification is necessary for the questions below.

(a) Let $s_n = \dfrac{3n + (-1)^n \cdot 2}{7n - (-1)^n \cdot 4}$.

- Does (s_n) converge or diverge? Converge

- If (s_n) converges, to what value does (s_n) converge? $3/7$

(b) Let $t_n = \dfrac{n}{5} + \dfrac{3}{n^2}$

- Does (t_n) converge or diverge? diverge

- If (t_n) converges, to what value does (t_n) converge? n/a