

## MATH 2300 – review problems for Exam 2

1. A metal plate of constant density  $\rho$  (in gm/cm<sup>2</sup>) has a shape bounded by the curve  $y = \sqrt{x}$ , the  $x$ -axis, and the line  $x = 1$ .
  - (a) Find the mass of the plate. Include units.
  - (b) Find the center of mass of the plate. Include units.
2. Write down (but do not evaluate) a definite integral representing the arc length of the function  $f(x) = \ln x$  between  $x = 1$  and  $x = 5$ .
3. If  $\int_1^5 f(x) dx = 7$  and  $\int_5^4 f(x) dx = 5$ , then find the average value of  $f(x)$  on  $[1, 4]$ .
4. Find the average value of  $f(x) = 3^{-x}$  on the interval  $[1, 3]$ .
5. pages 472-475: 7, 11, 15, 19, 31, 32, 49
6. Find the limit of all the sequences in the sequence activity:  
<http://math.colorado.edu/math2300/projects/SequencesPractice.pdf>
7. Does  $a_n = \frac{1}{n}$  converge? If so, what does it converge to?
8. Decide whether each of the following sequences converges. If a series converges, what does it converge to? If not, why not?
  - (a) The sequence whose  $n$ -th term is  $a_n = 1 - \frac{1}{n}$ .
  - (b) The sequence whose  $n$ -th term is  $b_n = \sqrt{n+1} - \sqrt{n}$ .
  - (c) The sequence whose  $n$ -th term is  $c_n = \cos(\pi n)$ .
  - (d) The sequence  $\{d_n\}$ , where  $d_1 = 2$  and

$$d_n = 2d_{n-1} \quad \text{for } n > 1.$$

9. Find the sum of the series. For what values of the variable does the series converge to this sum?
  - (a)  $1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} + \cdots$
  - (b)  $y - y^2 + y^3 - y^4 + \cdots$
  - (c)  $4 + z + \frac{z^2}{3} + \frac{z^3}{9} + \cdots$
10. For each of the following series, determine whether or not they converge. If they converge, determine what they converge to.

- (a)  $\sum_{n=1}^{\infty} 5 \left(\frac{2}{3}\right)^{n-1}$

- (b)  $\sum_{n=2}^{\infty} 3 \frac{4^{n+1}}{5^{n-4}}$

- (c)  $\sum_{n=3}^{\infty} \frac{7(-\pi)^{2n-1}}{e^{3n+1}}$

$$(d) \sum_{n=2}^8 4(.07)^{n+1}$$

$$(e) \sum_{n=1}^{\infty} \frac{1}{n^2 + 3n + 2}.$$

11. For each of the following series, determine if it converges absolutely, converges conditionally, or diverges. Completely justify your answers, including all details.

$$(a) \sum_{n=2}^{\infty} \frac{(-1)^n}{n+5}$$

$$(b) \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

$$(c) \sum_{n=2}^{\infty} \frac{n}{(\ln n)^2}$$

$$(d) \sum_{n=1}^{\infty} \frac{2n^2(-3)^n}{n!}$$

$$(e) \sum_{n=1}^{\infty} \left( \frac{4 \cdot 2^n}{(-3)^{n+1}} + \frac{1}{2^n} \right)$$

$$(f) \sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n^3 + 2n}$$

$$(g) \sum_{n=1}^{\infty} \frac{n + 3n^5}{2n^7 + 3}$$

$$(h) \sum_{n=1}^{\infty} n^{\frac{1}{n}}$$

$$(i) \sum_{n=1}^{\infty} \arctan n$$

$$(j) \sum_{n=1}^{\infty} \frac{\sin n}{n^2}$$

$$(k) \sum_{n=1}^{\infty} \left( \frac{n}{n+1} \right)^n$$

$$(l) \sum_{n=1}^{\infty} \frac{n!}{n^n}$$

12. Consider the series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

- (a) Confirm using the Alternate Series Test that the series converges.
- (b) How many terms must be added to estimate the sum to within .0001?
- (c) Estimate the sum to within .0001.

13. How many terms of  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$  should be added to estimate the sum to within .01? No calculators.

14. Check whether the following series converge or diverge. In each case, give the answer for convergence, and name the test you would use. If you use a comparison test, name the series  $\sum b_n$  you would compare to.

(a)  $\sum_{n=1}^{\infty} \frac{n}{(n^2 + 1)}$

(b)  $\sum_{n=1}^{\infty} \frac{n}{(n^2 + 1)^2}$

(c)  $\sum_{n=1}^{\infty} \left(n + \frac{1}{n}\right)^n$

(d)  $\sum_{n=1}^{\infty} \frac{\sqrt{n^4 + 1}}{5n^2}$

(e)  $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$  (hint: consider  $\sum_{n=1}^{\infty} \frac{1}{n^2}$  )

(f)  $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

(g)  $\sum_{n=1}^{\infty} \frac{(2n)!}{(n+3)!}$

(h)  $\sum_{n=1}^{\infty} \frac{n!}{(n+2)!}$

(i)  $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

15. Consider the series  $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ . Are the following statements true or false? Fully justify your answer.

(a) The series converges by limit comparison with the series  $\sum_{n=1}^{\infty} \frac{1}{n}$ .

(b) The series converges by the ratio test.

(c) The series converges by the integral test.

16. Consider the series  $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n}$ . Are the following statements true or false? Fully justify your answer.

(a) The series converges by limit comparison with the series  $\sum_{n=1}^{\infty} \frac{1}{n}$ .

- (b) The series converges by the ratio test.
- (c) The series converges by the integral test.
- (d) The series converges by the alternating series test.
- (e) The series converges absolutely.

17. Suppose the series  $\sum a_n$  is absolutely convergent. Are the following true or false? Explain.

- (a)  $\sum a_n$  is convergent.
- (b) The sequence  $a_n$  is convergent.
- (c)  $\sum (-1)^n a_n$  is convergent.
- (d) The sequence  $a_n$  converges to 1.
- (e)  $\sum a_n$  is conditionally convergent.
- (f)  $\sum \frac{a_n}{n}$  converges.

18. Does the following series converge or diverge?

$$\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$$

You must justify your answer to receive credit.

19. A ball is dropped from a height of 10 feet and bounces. Assume that there is no air resistance. Each bounce is  $\frac{3}{4}$  of the height of the bounce before.

- (1) Find an expression for the height to which the ball rises after it hits the floor for the  $n$ th time.
- (2) Find an expression for the total vertical distance the ball has traveled when it hits the floor for the  $n$ th time.
- (3) Using without proof the fact that a ball dropped from a height of  $h$  feet reaches the ground in  $\sqrt{h}/4$  seconds: Will the ball bounce forever? If not, how long it will take for the ball to come to rest?

Want more practice? Here's some more!

20. In theory, drugs that decay exponentially always leave a residue in the body. However in practice, once the drug has been in the body for 5 half-lives, it is regarded as being eliminated. If a patient takes a tablet of the same drug every 5 half-lives forever, what is the upper limit to the amount of drug that can be in the body?
21. Let  $\{f_n\}$  be the sequence defined recursively by  $f_1 = 5$  and  $f_n = f_{n-1} + 2n + 4$ .
- (a) Check that the sequence  $g_n$  whose  $n$ -th term is  $g_n = n^2 + 3n + 1$  satisfies this recurrence relation, and that  $g_1 = 5$ . (This tells us  $g_n = f_n$  for all  $n$ .)
- (b) Use the result of part (a) to find  $f_{20}$  quickly.
22. Find the values of  $a$  for which the series converges/diverges:

(a)  $\sum_{n=1}^{\infty} \left(\frac{1}{2a}\right)^n$

(b)  $\sum_{n=1}^{\infty} \frac{1}{a} \left(\frac{1}{2}\right)^n$

(c)  $\sum_{n=1}^{\infty} \left(\frac{2}{n}\right)^a$

(d)  $\sum_{n=1}^{\infty} (\ln a)^n$

(e)  $\sum_{n=1}^{\infty} \frac{1}{n(\ln n)^a}$

(f)  $\sum_{n=1}^{\infty} (1 + a^n)$

(g)  $\sum_{n=1}^{\infty} (1 + a)^n$

(h)  $\sum_{n=1}^{\infty} n^{\ln a}$

(i)  $\sum_{n=1}^{\infty} a^{\ln n}$

23. Using the table below, estimate the length of the curve given by  $y = f(x)$  from  $(3, 4)$  to  $(6, 0.7)$ .

$x$	3	3.5	4	4.5	5	5.5	6
$f(x)$	4	3.6	2.4	-1	-0.5	0	0.7
$f'(x)$	-0.8	-2.4	-6.8	1	1	1.4	-0.4

24. Determine if these sequences converge absolutely, converge conditionally or diverge.

(a)  $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$

(b)  $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 1}$

25. A steady wind blows a kite due east. The kite's height above ground from horizontal position  $x = 0$  to  $x = 80$  feet is given by

$$y = 150 - \frac{1}{40}(x - 50)^2$$

Find the distance traveled by the kite. Just set up the integral - don't evaluate.