### MATH 2300: Calculus II, Fall 2014 MIDTERM #2

Wednesday, October 15, 2014

#### YOUR NAME:

#### **Choose Your CORRECT Section**

Important note: SHOW ALL WORK. BOX YOUR ANSWERS. Calculators are not allowed. No books, notes, etc. Throughout this exam, please provide exact answers where possible. That is: if the answer is 1/2, do not write 0.499 or something of that sort; if the answer is  $\pi$ , do not write 3.14159.

Problem	Points	Score
1	10	
2	14	
3	12	
4	11	
5	15	
6	10	
7	18	
8	10	
TOTAL	100	

"On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work."

## SIGNATURE:

**1.** (2 points each) Match the recursively defined sequences in (a)-(e) to the sequences described in (I)-(V):

(a) $s_n = s_{n-1} + n$ for $n > 1$ and $s_1 = 1$	(I) $1, 3, 7, 15, \ldots$
(b) $s_n = s_{n-1} + \frac{1}{2}s_{n-2}$ for $n > 2, s_1 = 2, s_2 = 4$	(II) $2, 4, 7, 11, \ldots$
(c) $s_n = 2s_{n-1} + 1$ for $n > 1$ and $s_1 = 1$	(III) $1, 3, 6, 10, \dots$
(d) $s_n = (s_{n-1})^2 - n$ for $n > 2, s_1 = 1, s_2 = 3$	(IV) $1, 3, 6, 32, \dots$
(e) $s_n = s_{n-1} + n$ for $n > 1$ and $s_1 = 2$	(V) $2, 4, 5, 7, \ldots$

Write your answers below:

(8	u) (b	) (c)	) (d`	) (e	e)
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2. (14 points) Set up definite integrals computing the center of mass of an isosceles triangle whose density is  $\delta$  g/cm<sup>2</sup>, height is 8 cm, and base 4 cm. You do not need to solve the definite integrals.



**3.** (12 points) The region bounded by the curves  $y = x^4$  and y = x is rotated around the x-axis. Compute the volume of the solid generated.

# **4.** (11 points total)

(a) Find the Taylor polynomial of degree 2 approximating  $f(x) = \frac{2}{x+1}$  about x = 0.

(b) Find the Taylor polynomial of degree 2 approximating  $f(x) = \frac{2}{x+1}$  about x = 2.

5. (a) (3 points) Make a sketch of the cardioid  $r = 2 + 2\cos\theta$  and the circle r = 1 on the coordinate plane below. Shade the area inside the cardioid  $r = 2 + 2\cos\theta$  but outside the circle r = 1.



(b) (4 points) Calculate the polar coordinates of any points of intersection between  $r = 2 + 2\cos\theta$  and r = 1.

(c) (8 points) Set up an integral to compute the area inside  $r = 2 + 2\cos\theta$  but outside r = 1. You do not need to solve the integral.

6. (a) (5 points) Simplify  $-9 + 3 - 1 + \frac{1}{3} - \dots + \frac{1}{3^7}$ . Write the best option:

(I) 
$$9\left(\frac{1-(-1/3)^8}{1-(-1/3)}\right)$$
 (II)  $-9\left(\frac{1-(-1/3)^8}{1-(-1/3)}\right)$  (III)  $9\left(\frac{1-(-1/3)^9}{1-(-1/3)}\right)$   
(IV)  $-9\left(\frac{1-(-1/3)^9}{1-(-1/3)}\right)$  (V)  $-9\left(\frac{1-(-1/3)^{10}}{1-(-1/3)}\right)$  (VI)  $9\left(\frac{1-(-1/3)^{10}}{1-(-1/3)}\right)$ 

(b) (5 points) Find the sum of the geometric series  $8 - 2 + \frac{1}{2} - \frac{1}{8} + \frac{1}{32} + \cdots$ 

7. (6 points each) The following integrals represent the volume of either a sphere or a cone and the variable of integration measures length. In each case, **CIRCLE** which shape is represented and give the radius of the sphere or the radius and height of the cone. Make a sketch to support your answer showing the variable and all other relevant quantities.

(a) 
$$\int_{0}^{1} \pi (2x)^{2} dx$$
  
• SPHERE SKETCH:  
Radius =  
• CONE  
Radius =  
Height =  
(b) 
$$\int_{-4}^{4} \pi (16 - h^{2}) dh$$
  
• SPHERE SKETCH:  
Radius =  
• CONE  
Radius =  
Height =  
(c) 
$$\int_{0}^{5} \pi \left(\frac{5-y}{5}\right)^{2} dy$$
  
• SPHERE SKETCH:  
Radius =  
Height =  
• CONE  
Radius =  
Height =

8. (10 points total) No justification is necessary for the questions below.

(a) Let 
$$s_n = \frac{3n + (-1)^n \cdot 2}{7n - (-1)^n \cdot 4}$$
.

• Does  $(s_n)$  converge or diverge? \_\_\_\_\_

• If  $(s_n)$  converges, to what value does  $(s_n)$  converge?

(b) Let 
$$t_n = \frac{n}{5} + \frac{3}{n^2}$$

• Does  $(t_n)$  converge or diverge?

• If  $(t_n)$  converges, to what value does  $(t_n)$  converge?