5 f) $\displaystyle\int \frac{3x^2+6}{x^2(x^2+3)}\,dx$

Rational function - use partial fractions
degree of numerator is lower - good, no need
for long division.

$$\frac{3x^2+6}{x^2(x^2+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+3}$$

$$3x^2+6 = Ax(x^2+3) + B(x^2+3) + (Cx+D)x^2)$$

Substitute $x=0$: $6 = 3B$, $B=2$

Now look at coefficients:

$$3x^2+6 = Ax^3 + 3Ax + Bx^2 + 3B + Cx^3 + Dx^2$$

$$= (A+C)x^3 + (B+D)x^2 + 3Ax + 3B$$

Equate x^3 coefficients: $A+C=0 \Rightarrow A=-C$

Equate x^2 coefficients: $B+D=3 \Rightarrow D=1$

Equate x coefficients: $3A=0 \Rightarrow A=0 \Rightarrow C=0$

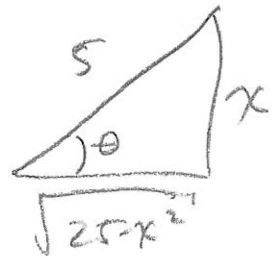
$$\int \frac{3x^2+6}{x^2(x^2+3)}\,dx = \int \frac{2}{x^2} + \frac{1}{x^2+3}\,dx$$

$$= -2x^{-1} + \frac{1}{\sqrt{3}}\arctan\frac{x}{\sqrt{3}} + c$$

5g) $\int \sqrt{25-x^2}\,dx$

trig substitution

$$\begin{cases} x = 5\sin\theta \\ dx = 5\cos\theta \\ \sqrt{25-x^2} = 5\cos\theta \\ \theta = \sin^{-1}\left(\frac{x}{5}\right) \end{cases}$$



$$= \int 5\cos\theta \cdot 5\cos\theta\, d\theta$$

$$= 25\int \cos^2\theta\, d\theta$$

$$= 25\int \frac{1+\cos 2\theta}{2}\, d\theta$$

$$= \frac{25}{2}\int 1 + \cos 2\theta\, d\theta$$

$$= \frac{25}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right] + c$$

$$= \frac{25}{2}\theta = \frac{25}{4}\cdot 2\sin\theta\cos\theta + c$$

$$= \frac{25}{2}\arcsin\left(\frac{x}{5}\right) + \frac{25}{2}\cdot\frac{x}{5}\cdot\frac{\sqrt{25-x^2}}{5} + c$$

$$= \frac{25}{2}\arcsin\left(\frac{x}{5}\right) + \frac{1}{2}x\sqrt{25-x^2} + c$$

7a. $\int_2^4 \frac{dx}{(x-3)^2}$ discontinuity at $x=3$, Wah! Split up into 2 improper integrals.

$$= \int_2^3 \frac{dx}{(x-3)^2} + \int_3^4 \frac{dx}{(x-3)^2}$$

Do the first one:

$$\int_2^3 \frac{dx}{(x-3)^2} = \lim_{b\to 3^-} \int_2^b \frac{dx}{(x-3)^2}$$

$$\lim_{b\to 3^-} -(x-3)^{-1}\Big|_2^b = \lim_{b\to 3^-} -\frac{1}{b-3} - 1$$

$$= +\infty$$

Diverges.

Since one of the two parts diverges, $\int_2^4 \frac{dx}{(x-3)^2}$ diverge

18 a) $\int_1^\infty \frac{\cos^2 x}{\sqrt{x^3}}\,dx.$ I don't know how to evaluate the integral, so I will compare

$0 < \cos^2 x \leq 1$

$0 \leq \frac{\cos^2 x}{x^{3/2}} \leq \frac{1}{x^{3/2}}$. $\int_1^\infty \frac{1}{x^{3/2}}\,dx$ converges.

(p-integral, with $p > 1$)

So by the comparison test, $\int_1^\infty \frac{\cos^2 x}{\sqrt{x^3}}$ also converges by the comparison theorem.

18d) $\displaystyle\int_3^\infty \frac{5 + 2\sin x}{x-2}\,dx$

$$-2 \le 2\sin x \le 2$$

$$3 \le 5 + 2\sin x \le 7$$

$$\frac{5+2\sin x}{x-2} \ge \frac{3}{x-2} > \frac{3}{x}$$

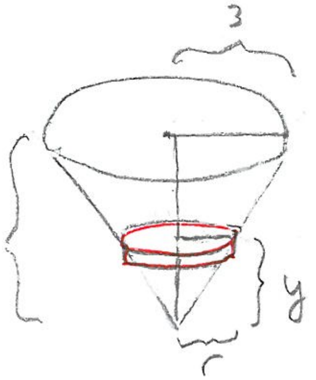
left side has a smaller denominator.

$\displaystyle\int_3^\infty \frac{3}{x}\,dx = 3\int_3^\infty \frac{1}{x}\,dx$ diverges $(p=1)$,

So $\displaystyle\int_3^\infty \frac{5+2\sin x}{x-2}\,dx$ also diverge

by the comparison theorem

20.



By similar triangles,

$\frac{10}{3} = \frac{y}{r}$

$10r = 3y$

$r = \frac{3}{10}y$

One slice 

Volume $= \pi r^2 h$

$= \pi r^2 \Delta y$

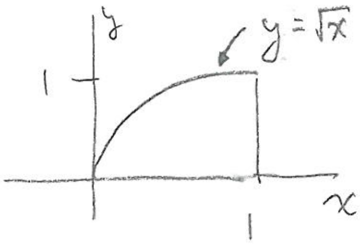
$= \pi \left(\frac{3}{10}y\right)^2 \Delta y$

Total volume $\int_0^{10} \pi \cdot \frac{9}{100} y^2 \, dy$

$= \frac{9\pi}{100} \left. \frac{y^3}{3} \right|_0^{10}$

$= \frac{9\pi}{100} \cdot \frac{1000}{3} = 90\pi$

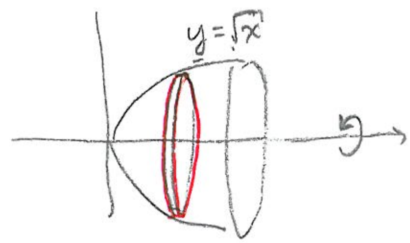
21.



a) area $= \int_0^1 \sqrt{x}\, dx$

$= \left. \frac{2}{3} x^{\frac{3}{2}} \right|_0^1 = \frac{2}{3}$

b)



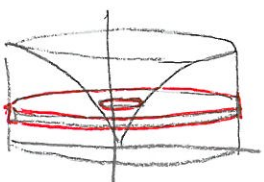
c) one slice:

$V = \pi r^2 h = \pi \cdot (\sqrt{x})^2 \cdot \Delta x$

$= \pi x \Delta x$

Total volume $= \int_0^1 \pi x \, dx = \left. \frac{\pi x^2}{2} \right|_0^1 = \frac{\pi}{2}$

d)



solve for x:

$x = y^2$

one slice:

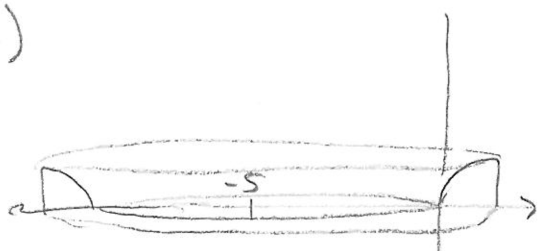
$\pi r_{outer}^2 \Delta y - \pi r_{inner}^2 \Delta y$

$= \pi \cdot 1^2 \Delta y - \pi (y^2)^2 \Delta y$

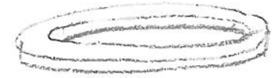
total volume $= \int_{y=0}^{y=1} \pi - \pi y^4 \, dy$

$= \left. \pi y - \frac{\pi y^5}{5} \right|_0^1 = \pi - \frac{\pi}{5} = \frac{4}{5}\pi$

e)



one slice



$V = (\pi r_{outer}^2 - \pi r_{inner}^2)\Delta y$

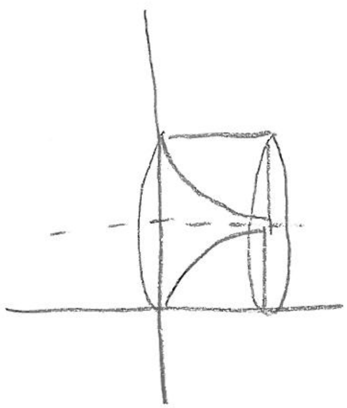
$r_{outer} = 1-(-5) = 6$

$= \pi\left(6^2 - (y^2-(-5))^2\right)\Delta y$

$r_{inner} = x-(-5) = y^2+5$

total volume $= \pi \displaystyle\int_{y=0}^{y=1} 36 - (y^2+5)^2\, dy$

f)



one slice:

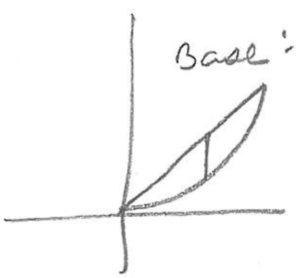
$V = (\pi r_{outer}^2 - \pi r_{inner}^2)\Delta x$

$= \pi\left[(1-0)^2 - (1-\sqrt{x})^2\right]\Delta x$

Total volume $= \pi \displaystyle\int_{x=0}^{x=1} 1 - (1-\sqrt{x})^2\, dx$

22.

Base:



cross-sections are perpendicular to the x-axis are squares.

$V = (x-x^2)^2\Delta x$

total volume $= \displaystyle\int_{x=0}^{x=1} (x-x^2)^2\, dx$