1. Determine the sum of the series $\sum_{n=1}^{+\infty} rac{2^n}{5^{n-2}}$

A)
$$\frac{2}{3}$$
 B) $\frac{10}{3}$ C) $\frac{20}{3}$ D) $\frac{40}{3}$ E) $\frac{50}{3}$

2. Determine the sum of the series
$$\sum_{n=1}^{+\infty} \frac{2^{n-1}-3^{n+1}}{4^n}$$

A)
$$-\frac{19}{6}$$
 B) $\frac{32}{3}$ C) $\frac{10}{3}$ D) $-\frac{17}{2}$ E) $-\frac{31}{9}$

3. For what value(s) of p does the series $\sum_{n=1}^{+\infty} \frac{n^{3p}+1}{\sqrt{n+9}}$ converge?

4. For what value(s) of a does the series
$$\sum_{n=1}^{+\infty} \left(rac{a}{n} - rac{1}{n+1}
ight)$$
 converge?

A) 0 B) 1 C) 2 D) All positive numbers E) No values

- 5. Given the series $\sum_{n=2}^{+\infty}a_n=\sum_{n=2}^{+\infty}rac{7n+3}{\sqrt{n^5-4}}$
 - (I), (II), and(III) provide the following arguments for convergence or divergence of the series.
 - (I): $b_n = \frac{n}{\sqrt{n^5}}, \quad \lim_{n \to \infty} \frac{a_n}{b_n} = 7, \quad \sum_{n=2}^{+\infty} a_n \text{ converges by Limit Comparison Test.}$
 - (II): $b_n=rac{7n}{\sqrt{n^5}}, \ \ 0\leq a_n\leq b_n, \ \ \sum_{n=2}^{+\infty}a_n$ converges by Comparison Test.
 - (III): $\lim_{n \to \infty} a_n \neq 0$, $\sum_{n=2}^{+\infty} a_n$ diverges by Test for Divergence.

Which argument(s) are correct?

A) (I) only B) (II) only C) (I) and (II) only D) (I), (II), and (III) E) None

6. Test the following series for convergence or divergence.

$$(I): \sum_{n=1}^{+\infty} \frac{\sqrt{n^3+1}}{3n^3+4n^2+2} \quad (II): \sum_{n=1}^{+\infty} ne^{-n^2} \quad (III): \sum_{n=1}^{+\infty} \sin\left(\frac{1}{n^3}\right)$$

- A) (I) converges, (II) converges, (III) converges
- B) (I) converges, (II) converges, (III) diverges
- C) (I) diverges, (II) converges, (III) converges
- D) (I) converges, (II) diverges, (III) converges
- E) (I) diverges, (II) converges, (III) diverges

7. A right inverted (i.e., upside down) circular conical tank of height 15 meters and base radius of 6 meters has its vertex at the bottom, and its axis vertical. If the tank is filled with water up to the 10-meter mark, then the work required to pump all the water out of the tank from the top is: (For computation, take the *y*-axis upwards along the axis of the tank and the origin at its vertex. The density of the water is 1000 kg/m^3 and the gravitational constant is $q \text{ m/s}^2$)

A)
$$1000g \int_0^{10} \pi \left(\frac{2y}{5}\right)^2 (15-y) \, dy$$

B) $1000g \int_0^{15} \pi \left(\frac{2y}{5}\right)^2 (10-y) \, dy$

C)
$$1000g \int_0^{10} \pi \left(\frac{2y}{5}\right)^2 y \, dy$$

D)
$$1000g \int_0^{15} \pi \left(\frac{2y}{5}\right)^2 y \, dy$$

E)
$$1000g \int_0^{10} \pi \left(\frac{2y}{5}\right)^2 (10-y) \, dy$$

8. Find the arc length of the curve $f(x) = rac{x^3}{3} + rac{1}{4x}$ on the interval [1,2]

A)
$$\frac{59}{24}$$
 B) $\frac{53}{24}$ C) $\frac{3\pi}{8}$ D) $\frac{3\pi}{4}$ E) $\arctan\left(\frac{13}{16}\right)$

9. Consider the lamina bounded by the curves $y = e^x$, y = 0, x = 0 and x = 1, with density $\rho = 1$. The y-coordinate \bar{y} of the center of mass of the lamina is

A)
$$\frac{1}{e-1}$$
 B) $e-1$ C) $e+1$ D) $\frac{e^2-1}{4}$ E) $\frac{e+1}{4}$

10. Determine whether the following sequences are convergent or divergent.

$$(I): \left\{a_n = \frac{(3n)!}{(5n)!}\right\} \ (II): \left\{a_n = n\sin\left(\frac{1}{n}\right)\right\} \ (III): \left\{a_n = n^2 e^{-n}\right\}$$

- A) (I) converges, (II) converges, (III) converges
- B) (I) converges, (II) converges, (III) diverges
- C) (I) diverges, (II) converges, (III) converges
- D) (I) converges, (II) diverges, (III) converges
- E) (I) diverges, (II) converges, (III) diverges

11. Evaluate $\lim_{n \to \infty} \sqrt[n]{2^n + 7^n + 9^n}$

12. A tank has the shape of the solid generated by revolving the region bounded by the graphs of $y = x^3$, x = 0, and y = 1 about the y-axis. The tank is full of water. Set up an integral for the work required to pump all the water to a level 4 feet above the top of the tank. (Water weighs 62.5 lbs/ft^3)

A)
$$62.5\pi \int_0^1 (4-y)y^{2/3} dy$$

B) $62.5\pi \int_0^1 (5-y)y^{2/3} dy$
C) $62.5\pi \int_0^4 (4-y)y^{1/3} dy$
D) $62.5\pi \int_0^5 (5-y)y^{2/3} dy$
E) $62.5\pi \int_0^4 (4-y)y^{2/3} dy$

 $13. \ \mbox{Determine}$ if the series converges or diverges. If it converges find its sum.

$$\sum_{n=2}^{+\infty} \frac{3^{n+2}}{4 \cdot 5^{2n-1}}$$

 $14. \ \mbox{Determine}$ if the series converges or diverges. If it converges find its sum.

$$\sum_{n=3}^{+\infty}rac{2}{n(n-1)}$$

 $15. \ \mbox{Determine}$ if the series converges or diverges.

$$\sum_{n=5}^{+\infty} \frac{\cos^{10}(n+5)}{\sqrt{n^3 + 4n^2 + 12n + 8}}$$

16. Find the average value of $f(x) = x \sin x$ over the interval $[0, \pi]$

17. A trough is filled with liquid of density 840 kg/m^3 . The ends of the trough are equilateral triangles with sides 8 meters long and vertex at the bottom. Find the hydrostatic force on one end of the trough.

18. Suppose a force of 10 N is required to stretch a spring 0.1 m from its equilibrium position and hold it in that position. How much work is needed to compress the spring 0.5 m from its equilibrium position?

A) 10.5 J B) 11.5 J C) 12.5 J D) 13.5 J E) 14.5 J

19. Determine if the series $\sum_{n=1}^{+\infty} \tan\left(rac{n+1}{n^3+4}
ight)$ converges or diverges.

20. Determine if the series
$$\sum_{n=2}^{+\infty} \frac{1}{n(\ln n)^3}$$
 converges or diverges.