# Midterm 2 – Math 2300 – November 13, 2013

On my honor as a University of Colorado at Boulder student I have neither given nor received unauthorized assistance on this exam.

**Name:**_____

Please select your section:

- ◯ **001** M. Grimes ..... (9 am)
- ◯ **002** A. Spina ..... (10 am)
- ◯ **003** A. Spina ..... (11 am)
- ◯ **004** C. Mesa ..... (12 noon)
- ◯ **005** F.T. Liu ..... (2 pm)
- ◯ **006** C. Bridges ..... (3 pm)
- ◯ **007** K. Smith ..... (8 am)
- ◯ **008** S. Henry ..... (2 pm)

In order to receive full credit your answer must be **complete**, **legible** and **correct**. You should show all of your work, and give clear explanations. **Calculators, phones or other electronic devices are not allowed.**

| Question | Points | Score |
|---|---|---|
| 1 | 20 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 18 | |
| 5 | 10 | |
| 6 | 12 | |
| 7 | 12 | |
| 8 | 8 | |
| Total: | 100 | |

1. (20 points) Determine if the following series converge absolutely, converge conditionally or diverge. Give complete explanations.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

2. (10 points) Find the interval of convergence for the following series. Show all of your work and explain your reasoning.

$$\sum_{n=1}^{\infty}(-1)^n \frac{(x-2)^n}{n}$$

3. (10 points) Use a $3^{rd}$ degree Taylor polynomial to estimate $\ln(1.1)$. Show your work. (Do not simplify your answer.)

4. (18 points) Use any method you like to find Taylor series about $a = 0$ for the following functions.

(a) $f(x) = \sin(x^2)$

(b) $g(x) = xe^x$

(c) $k(x) = \frac{1}{(1-x)^2}$

5. (10 points) The Taylor series for $\arctan(x^2)$ is given by

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{4n+2}.$$

Is it possible to use this Taylor series to approximate $\arctan(4)$? Justify your answer.

6. (12 points) Use the Lagrange Error Bound to find a bound on the error in using a 4th degree Taylor Polynomial for $f(x) = e^x$ about $a = 0$ to estimate $\sqrt{e}$.

7. (12 points) For what values of $k$ (if any) does $y = 5 + 3e^{kx}$ satisfy the differential equation $\frac{dy}{dx} = 10 - 2y$?

8. (8 points) For each of the following, circle True or False

(a) **True** **False** If $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=1}^{\infty} a_n$ must converge.

(b) **True** **False** If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then $\sum_{n=1}^{\infty} \frac{a_n}{n}$ must converge.

(c) **True** **False** If $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} |a_n|$ must converge.

(d) **True** **False** If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{2}$, then $\lim_{n\to\infty} a_n = 0$.