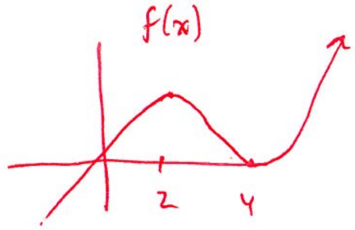
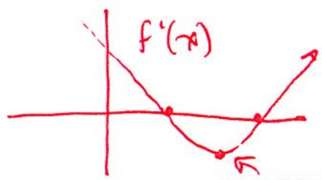
REVIEW SOLUTIONS

1.





corresponds to inflection point of f

2. $\lim_{x\to\infty} xe^{-x} = \lim_{x\to\infty} \frac{x}{e^x}$ "$\frac{\infty}{\infty}$"

$\overset{L'Ho}{=} \lim_{x\to\infty} \frac{1}{e^x} = 0$

3. $f(x) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$

c.p's at $x=0$, $x=3$

$f'(x)$: $-$ (0) $-$ (3) $+$

f increases on $(3,\infty)$ and decrease on $(-\infty, 3)$

$f''(x) = 12x^2 - 24x$

$= 12x(x-2)$

$f''(x) = 0$ at $x=0$, $x=2$

$f''(x)$: $+$ (0) $-$ (2) $+$

f concave up on $(-\infty,0)\cup(2,\infty)$, concave down on $(0,2)$

4. $a(t) = -4$

$v(t) = -4t + C$ "Dropped" means when $t=0$, $v=0$

$0 = -4\cdot 0 + C \Rightarrow C = 0$

$v(t) = -4t$

$s(t) = -2t^2 + C_2$ when $t=0$, $s=16$

$16 = -2\cdot 0 + C_2 \Rightarrow C_2 = 16$

$s(t) = -2t^2 + 16$

"hits ground" means $s=0 \Rightarrow -2t^2+16=0$

$2t^2 = 16 \Rightarrow t^2 = 8 \Rightarrow t = \sqrt{8}$ sec.

5. $\frac{1}{3-1}\int_1^3 \frac{\sqrt{x}+x+1}{x^2}\,dx$

$= \frac{1}{2}\int_1^3 (\sqrt{x}+x+1)x^{-2}\,dx = \frac{1}{2}\int_1^3 x^{-3/2} + \frac{1}{x} + x^{-2}\,dx$

$= \frac{1}{2}\left(-2x^{-\frac{1}{2}} + \ln|x| - x^{-1}\right)\Big|_1^3$

$= \frac{1}{2}\left(-2\cdot 3^{-\frac{1}{2}} + \ln 3 - \frac{1}{3}\right) - \frac{1}{2}\left(-2+\ln 1 - 1\right)$

$= \frac{1}{2}\left(-\frac{2}{\sqrt{3}} + \ln 3 - \frac{1}{3}\right) - \frac{1}{2}(-3)$

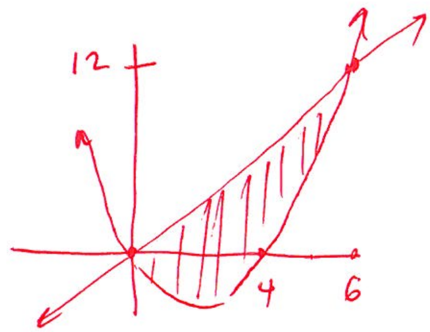
6. $f(x) = 2x,\quad f(x) = x^2 - 4x$

intersection: $x^2 - 4x = 2x$

$x^2 - 6x = 0$

$x(x-6) = 0$

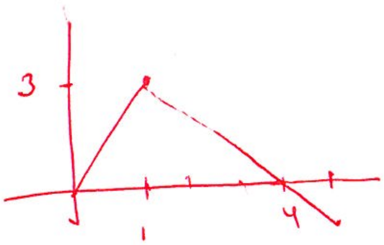
$x = 0,\ x = 6$



area $= \int_0^6 2x - (x^2 - 4x)\,dx$

$= \int_0^6 6x - x^2\,dx = 3x^2 - \frac{1}{3}x^3\Big|_0^6 = 108 - 72 = 36$

7.



$f(x)$ shown

$F(x) = \int_1^x f(t)\,dt$

$F'(x) = f(x)$.

$F(1) = 0$

$F(4) = \frac{9}{2}$

$F(5) = 4$

$F(0) = -\frac{3}{2}$

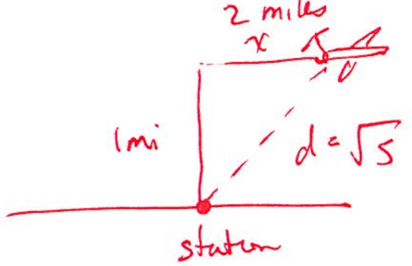
inc. on $(0,4)$

dec on $(4,5)$

conc. up on $(0,1)$

conc. down on $(1,5)$

8.



Given: $\frac{dx}{dt} = 500 \frac{\text{mile}}{\text{hour}}$

want: $\frac{dd}{dt}$.

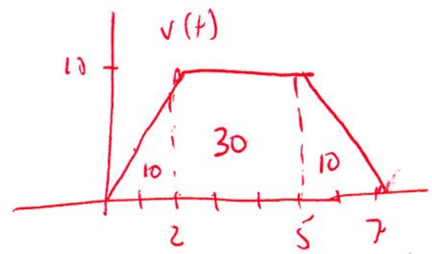
$$x^2 + 1 = d^2$$

$$2x\frac{dx}{dt} + 0 = 2d\frac{dd}{dt}$$

$$\cancel{2} \cdot 2 \cdot 500 = \cancel{2} \cdot \sqrt{5} \cdot \frac{dd}{dt}$$

$$\frac{dd}{dt} = \frac{1000}{\sqrt{5}} \frac{\text{miles}}{\text{hour}}.$$

9. $s(x) = s(2) + \int_2^x v(t)\,dt$



$s(2) = 8$ (given)

$s(0) = 8 + \int_2^0 v(t)\,dt = 8 - 10 = -2$

$s(1) = 8 + \int_2^1 v(t)\,dt = 8 - 7.5 = .5$

$s(4) = 8 + 20 = 28$

$s(7) = 8 + 30 + 10 = 48$

$a(t) = v'(t)$, slope of velocity curve

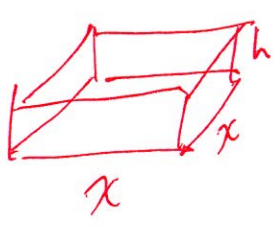
$a(1) = \frac{10}{2} = 5$

$a(4) = 0$

$a(6) = \frac{-10}{2} = -5$

10. SEE REVIEW SHEET FOR EXAMPLES

11.



Surface area $= 2x^2 + 4xh = 1000$, so $h = \frac{1000 - 2x^2}{4x}$

$V = x^2 h = x^2\left(\frac{1000 - 2x^2}{4x}\right) = \frac{1000}{4}x - \frac{2}{4}x^3$

$V'(x) = 250 - \frac{3}{2}x^2 = 0 \Rightarrow x = \sqrt{\frac{500}{3}}$

domain: $(0, \sqrt{500})$

V has a local max at its only c.p.

V also has a global max there

$V = \left(\sqrt{\frac{500}{3}}\right)^2 \left(\frac{1000 - 2 \cdot 500/3}{4 \cdot \sqrt{500/3}}\right)$ (This simplifies)

12. a. $f(x) = (\arctan x)^{1/2}$

$f'(x) = \frac{1}{2}(\arctan x)^{-1/2} \cdot \frac{1}{1+x^2}$

b. $f(x) = \int_{-x}^{x^3} \tan t \, dt$

$f'(x) = \tan(x^3) \cdot 3x^2 - \tan(-x)(-1)$

c. $f(x) = \frac{x}{\ln x}$

$f'(x) = \frac{\ln x - x \cdot \frac{1}{x}}{(\ln x)^2}$

13. a) $\int \sec^2 x \, dx = \tan x + C$

b) $\int_0^2 x e^{x^2} dx$
$\begin{cases} u = x^2 \\ du = 2x\,dx \\ x=0 \Rightarrow u=0 \\ x=2 \Rightarrow u=4 \end{cases}$

$= \frac{1}{2}\int_0^2 2x e^{x^2} dx$

$= \frac{1}{2}\int_0^4 e^u du$

$= \frac{1}{2} e^u \Big|_0^4 = \frac{1}{2}(e^4 - 1)$

c) $\int 10^x dx = \frac{1}{\ln 10} 10^x + C$

d) $\int_0^1 \frac{1}{1+x^2} dx = \arctan x \Big|_0^1 = \arctan 1 - \arctan 0 = \frac{\pi}{4}$

e) $\int_0^1 \frac{x}{1+x^2} dx$
$u = 1+x^2$
$du = 2x\,dx$

$= \frac{1}{2}\int_0^1 \frac{2x}{1+x^2} dx$

$= \frac{1}{2}\int_1^2 \frac{1}{u} du$

$= \frac{1}{2} \ln|u| \Big|_1^2 = \frac{1}{2}\ln 2 - \frac{1}{2}\ln 1 = \frac{1}{2}\ln 2$

14. $$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

If $f(x) = 3x^2+4x$

$$f'(x) = \lim_{h\to 0} \frac{3(x+h)^2 + 4(x+h) - (3x^2+4x)}{h}$$

$$= \lim_{h\to 0} \frac{3(x^2+2xh+h^2) + 4x + 4h - 3x^2 - 4x}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 + \cancel{4x} + 4h \cancel{-3x^2} \cancel{-4x}}{h}$$

$$= \lim_{h\to 0} \frac{6xh + 3h^2 + 4h}{h} = \lim_{h\to 0} \frac{\cancel{h}(6x+3h+4)}{\cancel{h}}$$

$$= \lim_{h\to 0} 6x+3h+4 = 6x+4$$

Note: $\frac{d}{dx}(3x^2+4x) = 6x+4$, yup, matches our rules of differentiation!

15. Estimate $\sqrt[3]{8.3}$ using linear approximation.

$f(x) = x^{1/3}$ $\quad f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

$a=8$ $\quad f(8)=2$ $\quad f'(8) = \frac{1}{3}\cdot 8^{-\frac{2}{3}} = \frac{1}{3}\cdot\frac{1}{4} = \frac{1}{12}$

$L(x) = f(a) + f'(a)(x-a)$

$= 2 + \frac{1}{12}(x-a)$

$x = 8.3$

$L(8.3) = 2 + \frac{1}{12}(8.3-8)$

$= 2 + \frac{1}{12}(.3) = 2.025$