Review Sheet for Third Midterm
Mathematics 1300, Calculus 1

1. For $f(x) = x^3 - 3x^2$ on $-1 \leq x \leq 3$, find the critical points of f, the inflection points, the values of f at all these points and the endpoints, and the absolute maxima and minima of f. Then sketch the graph, indicating clearly where f is increasing or decreasing and its concavity.

2. For $f(x) = x + \sin x$ on $0 \leq x \leq 2\pi$, find the critical points of f, the inflection points, the values of f at all these points and the endpoints, and the absolute maxima and minima of f. Then sketch the graph, indicating clearly where f is increasing or decreasing and its concavity.

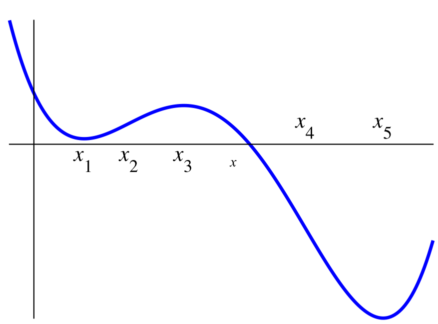
3. For $f(x) = \dfrac{4x^2}{x^2+1}$, find the critical points of f, the inflection points, the values of f at all these points, the limits as $x \to \pm\infty$, and the absolute maxima and minima of f. Then sketch the graph, indicating clearly where f is increasing or decreasing and its concavity.

4. Find the exact absolute maximum and minimum values of the function
$$h(z) = \frac{1}{z} + 4z^2 \text{ for } z > 0$$

5. Use derivatives to identify local maxima and minima and points of inflection. Then graph the function.
$$f(x) = e^{-x^2}.$$

6. (a) Find all critical points and all inflection points of the function $f(x) = x^4 - 2ax^2 + b$. Assume a and b are positive constants.
   (b) Find values of the parameters a and b if f has a critical point at the point $(2, 5)$.
   (c) If there is a critical point at $(2, 5)$, where are the inflection points?

7. For the function, f, graphed in the Figure:



   (a) Sketch $f'(x)$.

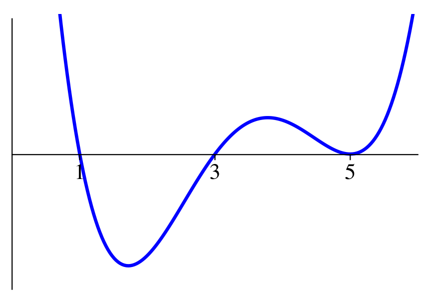
(b) Where does $f'(x)$ change its sign?

(c) Where does $f'(x)$ have local maxima or minima?

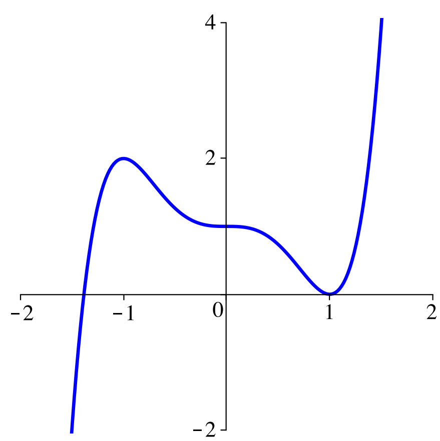
8. Using your answer to the previous problem as a guide, write a short paragraph (using complete sentences) which describes the relationships between the following features of a function f:

- The local maxima and minima of f.
- The points at which the graph of f changes concavity.
- The sign changes of f'.
- The local maxima and minima of f'.

9. The figure shows a graph of $y = f'(x)$ (not the function f). For what values of x does f have a local maximum? A local minimum?

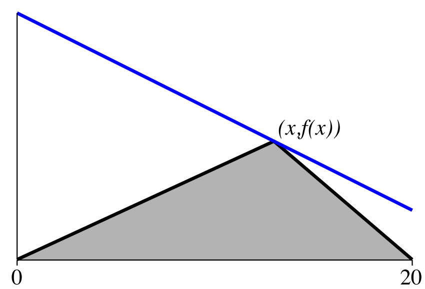


10. On the graph of f' shown in the figure, indicate the x-values that are critical points of the function f itself. Are they local minima, local maxima, or neither?



11. Find a formula for the function described: a cubic polynomial with a local maximum at $x = 1$, a local minimum at $x = 3$, a y-intercept of 5, and an x^3 term whose coefficient is 1.

12. Find the x-value maximizing the shaded area. One vertex is on the graph of $f(x) = x^2/3 - 50x + 1000$ (shown in blue).



13. Given that the surface area of a closed cylinder of radius r cm and height h cm is 8 cm^2, find the dimensions giving the maximum volume.

14. Do the last two problems from Project 11.

15. Find the range of the function $f(x) = x^3 - 6x^2 + 9x + 5$, if the domain is $0 \le x \le 5$.

16. A landscape architect plans to enclose a 3000 square foot rectangular region in a botanical garden. She will use shrubs costing \$45 per foot along three sides and fencing costing \$20 per foot along the fourth side. Find the minimum total cost.

17. A piece of wire of length L cm is cut into two pieces. One piece, of length x cm, is made into a circle; the rest is made into a square. Find the value of x that makes the sum of the areas of the circle and square a minimum. Find the value of x giving a maximum.

18. Does L'Hopital's rule apply to the limit? If so, evaluate it.

$$\lim_{x \to 0} \frac{x}{e^x}.$$

19. Does L'Hopital's rule apply to the limit? If so, evaluate it.

$$\lim_{x \to 1} \frac{\sin \pi x}{x - 1}.$$

20. Determine whether the limit exists, and if possible evaluate it.

$$\lim_{t \to 0} \frac{e^t - 1 - t}{t^2}$$

21. Determine whether the limit exists, and if possible evaluate it.

$$\lim_{t \to 0^+} \frac{3 \sin t - \sin 3t}{3 \tan t - \tan 3t}$$

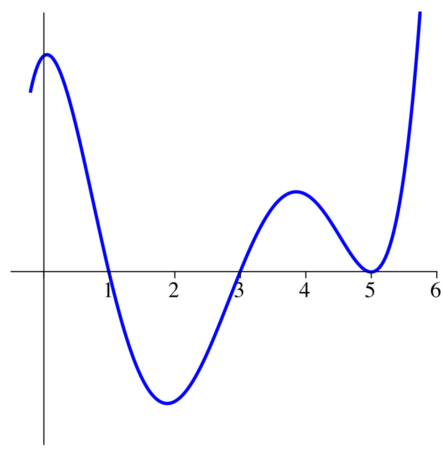
22. True or false? If true, explain how you know. If false, give a counterexample. If $f'(p) = 0$, then $f(x)$ has a local minimum or local maximum at $x = p$.

23. True or false? If true, explain how you know. If false, give a counterexample. If $f''(p) = 0$, then the graph of f has an inflection point at $x = p$.

24. Sketch the graph of a function defined at every point of $[0, 2]$ which has an absolute minimum but no absolute maximum. Why does your graph not contradict the Extreme Value Theorem?

25. Precisely state the Mean Value Theorem.

26. The graph of $y = f''(x)$ is shown. Where are the inflection points of $y = f(x)$?



27. Find the absolute maximum and minimum of the function $f(x) = 9x^{1/3} - x^3$ on the closed interval $[-1, 8]$.

28. Which point on the parabola $y = x^2 - x$ minimizes the square of the distance to the point $(1, 1)$?

29. What is the largest area a rectangle with a perimeter of 40 inches can have?

30. You have 80 feet of fencing and want to enclose a rectangular area up against a long, straight wall (using the wall for one side of the enclosure and the fencing for the other three sides of the enclosure). What is the largest area you can enclose?

31. Compute
$$\lim_{n\to\infty} \left(1 + \frac{2}{n}\right)^n.$$

32. Compute
$$\lim_{x\to 1} \frac{\ln x}{x^2 - 1}.$$

33. Compute

$$\lim_{x\to 2} \frac{x^2 - 3x + 2}{x + 5}.$$

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34. Let f be a continuous function with domain $[-1, 3]$, and assume that f is differentiable on $(-1, 3)$. Which of the following statements about f are definitely true?

(a) If f has a critical point at $x = 2$, then f has either a local maximum or a local minimum at $x = 2$.

(b) If f has a critical point at $x = 1$ and $f''(x) < 0$, then f has a local maximum at $x = 1$.

(c) If f has a critical point at $x = 1$ and $f''(x) < 0$, then f has an absolute maximum at $x = 1$.

(d) If $f(3) > f(-1)$, then f has an absolute maximum at $x = 3$.

(e) If $f'(0) = 0$ and $f(0) > f(3) > f(-1)$, then f has an absolute maximum at $x = 0$.

(f) If the only critical point of f occurs at $x = 2$, then f must have an absolute maximum at either $x = -1, 2$, or 3.

(g) It is possible that f might not have an absolute minimum on this domain.

(h) If f has a local minimum at $x = 2$ then $f'(2) = 0$.

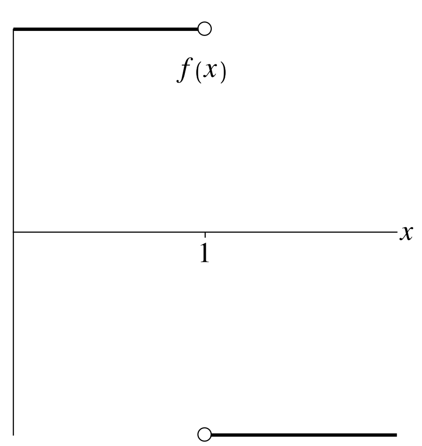
(i) If $f''(x) > 0$ for all values of x in the domain, then $f(3) \geq f(-1)$.

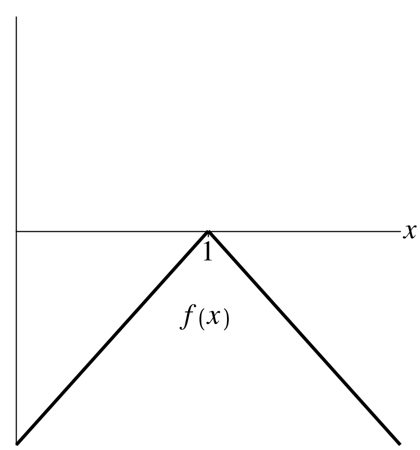
(j) If $f''(x) > 0$ for all values of x in the domain, then $f'(3) \geq f'(-1)$.

(k) If f has an inflection point at $x = 1$ then $f'(x)$ has either a local maximum or a local minimum at $x = 1$.

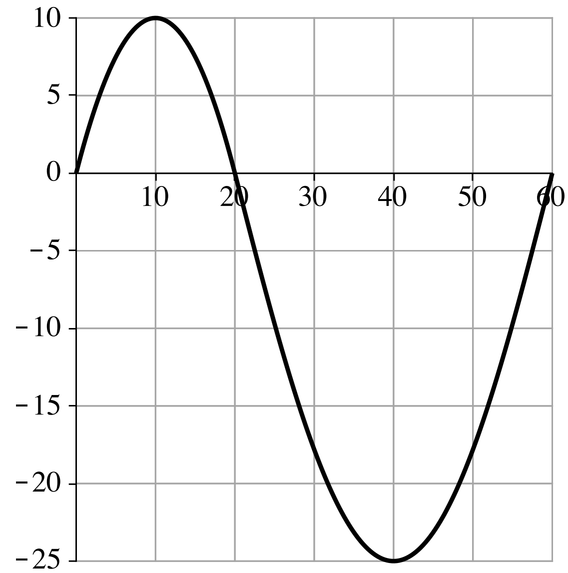
(l) If $f''(1) = 0$ then f has an inflection point at $x = 1$.

35. For each of the graphs $f(x)$ shown below, sketch a graph of $F(x)$ such that $F'(x) = f(x)$ and $F(0) = 1$.





36. A bicyclist is pedaling along a straight road for one hour with a velocity v shown in the figure. She starts out five kilometers from the lake and positive velocities take her toward the lake. Note that the horizontal units are in minutes while the vertical units are in kilometers per hour.



(a) Does the cyclist ever turn around? If so, at what time(s)?

(b) When is she going the fastest? How fast is she going then? Toward the lake or away?

(c) When is she closest to the lake? Approximately how close to the lake does she get?

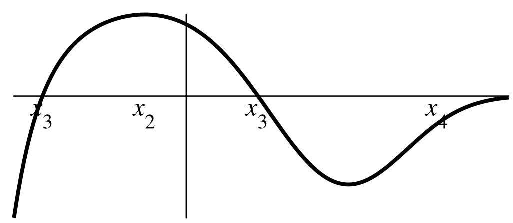
(d) When is she farthest from the lake? Approximately how far from the lake is she then?

37. Suppose that $f(2) = 4$, and that the table below gives values of f' for x in the interval $[0, 12]$

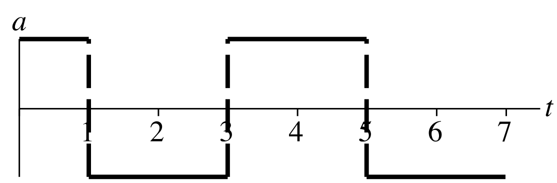
x	0	2	4	6	8	10	12
$f'(x)$	-19	-21	-25	-28	-29	-28	-25

Estimate $f''(2)$, and estimate $f(8)$.

38. The graph of $f'(x)$ is given. Sketch a possible graph for $f(x)$. Mark the points $x_1, \ldots, x_4$ on your graph and label local maxima, local minima, and inflection points on your graph. Note, the left most x_3 in the figure should be x_1.



39. The acceleration, a, of a particle as a function of time is shown in the figure. Sketch graphs of velocity and position against time. The particle starts at rest at the origin.



40. An old rowboat has sprung a leak. Water is flowing from the boat at a rate, $r(t)$, given in the following table.

t minutes	0	5	10	15
$r(t)$ liters/min	12	20	24	16

(a) Compute upper and lower estimates for the volume of water that has flowed into the boat during the 15 minutes.

(b) Draw a graph to illustrate the lower estimate.

41. Find the antiderivative of $f(x) = 7x^6 - 4x^2 + x$.

42. Find the antiderivative of $f(x) = 2e^x - 3\sec^2 x$.

43. Find the antiderivative of $f(x) = 2^x + x^{-\frac{1}{2}} + \sin x$.

44. Find the antiderivative of $f(x) = \frac{18x^6+6x^5+3x^3}{x^6}$.

45. Find the antiderivative F of $f(x) = \sec x \tan x$ such that $F(0) = 101$.

46. Find the antiderivative F of $f(x) = 11e^x - \frac{1}{x}$ such that $F(1) = e$.

47. Find f if $f''(x) = \cos x + x^5$.