Some Review Problems for First Midterm Mathematics 1300, Calculus 1

1. Consider the trigonometric function f(t) whose graph is shown below. Write down a possible formula for f(t).



- 2. Find the domain of the function $f(x) = \sqrt{4-5x}$.
- 3. What is the domain and range of the function $f(x) = (x+3)^2 4$?
- 4. Use the limit definition of derivative to compute the derivative of the function $f(x) = \frac{4}{x}$ at x = 2.
- 5. The table below gives the depth of snow that has fallen in inches as a function of time in hours past 8am. What is d(1.5) (including units) and what does it represent? What time did it start snowing? Next, estimate d'(1) and d'(2). Include units in your answer and say in a full English sentence what the meaning of these numbers are.

| x | .75 | 1 | 1.25 | 1.5 | 1.75 | 2 | 2.25 | 2.5 |
|------|-----|----|------|------|------|-----|------|-----|
| d(x) | 0 | .3 | .8 | 1.25 | 1.85 | 2.0 | 2.2 | 2.4 |

- 6. Say $f(x) = 3x^2 + x$. Use the definition of derivative to find f'(1). Then write the equation of the tangent line to the curve at x = 1.
- 7. Say that the position of an object moving horizontally is given by $s(t) = \sqrt{6t+7}$, where position is measured in miles and time is measured in hours. Find the instantaneous velocity at an arbitrary t = a, and then the instantaneous velocity at t = 7. Include units.
- 8. Consider the function

$$f(x) = \begin{cases} x^2 + \cos x & \text{when } -5 < x < 0\\ 5 & \text{when } x = 0\\ e^{3x/2} & \text{when } 0 < x < 5 \end{cases}$$

(a) Define what it means for a function f(x) to be continuous at a point x = a.

- (b) $\lim_{x \to 2^+} f(x) =$
- (c) $\lim_{x \to 0^+} f(x) =$
- (d) $\lim_{x \to 0^{-}} f(x) =$
- (e) $\lim_{x \to 0} f(x) =$
- (f) Is f(x) continuous at x = 0? Fully explain your answer. If it is not continuous, state what type of discontinuity it is and why.
- 9. Sketch the graph of a function g(x) that satisfies ALL of the following properties:
 - (a) $\lim_{x \to 1^+} g(x) = 2$
 - (b) $\lim_{x \to 1} g(x)$ does not exist.
 - (c) $\lim_{x \to \infty} g(x) = -4$
 - (d) $\lim_{x \to -4^+} g(x) = -\infty$

(e)
$$\lim_{x \to -\infty} g(x) = \infty$$

- 10. A car is first driven at an increasing speed and then the speed begins to decrease until the car stops. Sketch a graph of the distance the car has traveled as a function of time.
- 11. Find the average velocity over the interval $0.2 \le t \le 0.3$ of a car whose position s(t) is given by the following table. Then estimate the velocity at t = 0.3. Include units.

| $t \; (sec)$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|--------------|-----|-----|-----|-----|-----|
| s(t) (ft) | 0 | 0.5 | 1.0 | 1.8 | 2.8 |

- 12. The function $f(t) = -16t^2 + 64t$ gives the distance above the ground of a ball that is thrown from ground level straight up into the air at time t = 0, with an initial velocity of 64 ft/sec.
 - (a) How high is the ball above the ground at t = 1 second?
 - (b) How fast is the ball moving at t = 1, t = 2, t = 3 and t = 4 seconds?
 - (c) What is the maximum height of the ball? (Hint: you should have gotten a velocity of 0 for one of the values of t in the last part)
- 13. Precisely state the Intermediate Value Theorem. A polynomial p(x) has p(-2) = 13 and p(0) = -1 and p(1) = 1. Show that p(x) has at least two zeroes.
- 14. Find the following limits. Show all of your work.

(a)
$$\lim_{x \to 2} \frac{x^2 + 4x - 12}{x - 2}$$

(b)
$$\lim_{x \to 9} \frac{3 - \sqrt{x}}{x - 9}$$

(c)
$$\lim_{x \to 4} \frac{\frac{2}{x} - \frac{1}{2}}{x - 4}$$

(d)
$$\lim_{x \to \infty} \frac{3x^4 + 2x - 1}{4 - x^4}$$

(e)
$$\lim_{x \to \infty} \frac{3x^2 + 2x - 1}{4 - x}$$

(f)
$$\lim_{x \to \infty} \frac{\sqrt{4x^2 + 2x - 1}}{x + 1}$$

(g)
$$\lim_{x \to -\infty} (\sqrt{9x^2 + x} - 3x)$$

(h)
$$\lim_{x \to \infty} (\sqrt{9x^2 + x} - 3x)$$

15. The curve below shows position s, measured in feet, as a function of time t, measured in seconds. The dotted line is tangent to the curve.



- (a) Find the average velocity over the interval from t = 0 to t = 3. Include units.
- (b) Find the instantaneous velocity at t = 2. Include units.
- (c) Find the equation of the tangent line.

16. Here's a graph of the function f(x):



Meanwhile, here is a definition of the function g(x):

$$g(x) = \begin{cases} -(x-3)^2 + 1 & x > 3 \\ -x+6 & x < 3 \end{cases}$$

- (a) Explain why the graph defines f as a function of x.
- (b) Is the f function invertible?
- (c) Is the function g invertible?
- (d) What is the domain and the range of f?
- (e) What is the domain and the range of g?
- (f) Find the limit of f(x) as x approaches -2, as x approaches 1, as x approaches 2, as x approaches 3, as x approaches 4. Clearly explain your results.
- (g) What is the limit of g(x) as x approaches 3? What about as x approaches 0?
- (h) List all of the discontinuities of f and g and clearly explain why these are discontinuities. State what type of discontinuities they are and why.
- (i) $\lim_{x \to 2} (f(x) + g(x)) =$

(j)
$$\lim_{x \to 3} (f(x) + g(x)) =$$

- 17. Find two values of the constant b so that the following function is continuous: $h(x) = \begin{cases} x^2 + x - 11 & x \ge b \\ -3x + 1 & x < b \end{cases}$
- 18. If $4x 9 \le f(x) \le x^2 4x + 7$ for $x \ge 0$, then find $\lim_{x \to 4} f(x)$.
- 19. Suppose f(2) = 3 and f'(2) = 1. Find f(-2) and f'(-2) if f is assumed to be even.
- 20. Given all of the following information about a function f, sketch its graph.
 - f(x) = 0 at x = -5, x = 0, and x = 5
 - $\lim_{x \to -\infty} f(x) = \infty$
 - $\lim_{x \to \infty} f(x) = -3$
 - f'(x) = 0 at x = -3, x = 2.5, and x = 7
- 21. Given the graph of y = f(x) shown, sketch the graph of the derivative.



22. Given the following table of values, estimate f'(0.6) and f'(0.5); then use these to estimate f''(0.6).

| x | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|------|-----|-----|-----|-----|-----|-----|
| f(x) | 3.7 | 3.5 | 3.5 | 3.9 | 4.0 | 3.9 |

- 23. If possible, draw the graph of a differentiable function with domain [0,6] satisfying f'(x) > 0 for x < 1, f'(x) < 0 for x > 1, f'(x) > 0 for x > 4, f''(x) > 0 for x < 3, and f''(x) < 0 for x > 3.
- 24. If y = f(x) is the graph shown below, sketch the graphs of y = f'(x) and y = f''(x). Estimate slopes as needed.



25. The graph of y = f'(x) is shown. Where is f(x) greatest? Least? Where is f'(x) greatest? Least? Where is f''(x) greatest? Least? Where is f(x) concave up? Concave down?



26. Name three different reasons a function can fail to be differentiable at a point.