1. Name three different reasons that a function can fail to be differentiable at a point. Give an example for each reason, and explain why your examples are valid.

   Solution: It could be discontinuous, or have a vertical tangent like $y = x^{1/3}$, or have a corner like $y = |x|$.

2. Given the following table of values, estimate $f'(0.6)$ and $f'(0.5)$; then use these to estimate $f''(0.6)$.

x	0	0.2	0.4	0.6	0.8	1.0
$f(x)$	3.7	3.5	3.5	3.9	4.0	3.9

   Solution: $f'(0.6) \approx 1.25$ and $f'(0.5) \approx 2$, so $f''(0.6) \approx -7.5$.

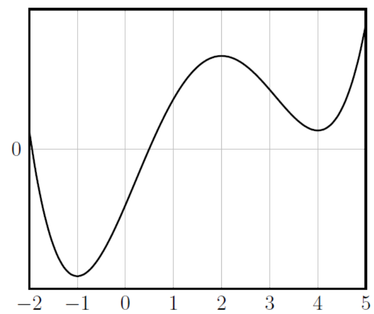
3. If possible, draw the graph of a differentiable function with domain $[0, 6]$ satisfying $f'(x) > 0$ for $x < 1$, $f'(x) < 0$ for $x > 1$, $f'(x) > 0$ for $x > 4$, $f''(x) > 0$ for $x < 3$, and $f''(x) < 0$ for $x > 3$.

   Solution: It's impossible. If $f(x)$ is differentiable at $x = 1$, then the fact that $f''(x) > 0$ for $x < 3$ means that $f'(x)$ is increasing, so $f'(x)$ is increasing, so $f'(x)$ can't go from positive on $[0, 1)$ to negative on $(1, 4)$.

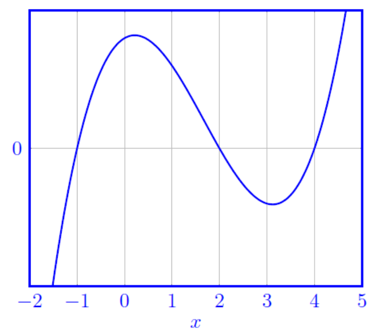
4. Given all of the following information about a function f sketch its graph.

   - $f(x) = 0$ at $x = -5, x = 0$, and $x = 5$
   - $\lim_{x\to-\infty} f(x) = \infty$
   - $\lim_{x\to\infty} f(x) = -3$
   - $f'(x) = 0$ at $x = -3, x = 2.5$, and $x = 7$

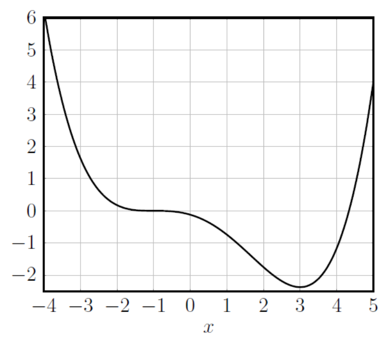
5. Given the graph of $y = f(x)$ shown, sketch the graph of the derivative.



Solution:

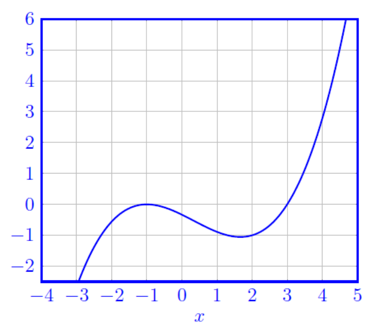


6. If $y = f(x)$ is the graph shown below, sketch the graphs of $y = f'(x)$ and $y = f''(x)$. Estimate slopes as needed.

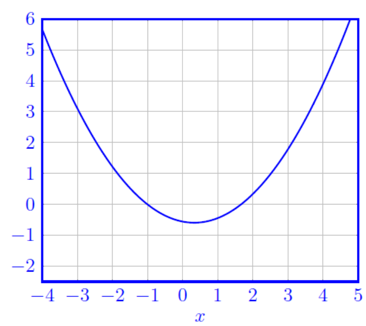


Solution:

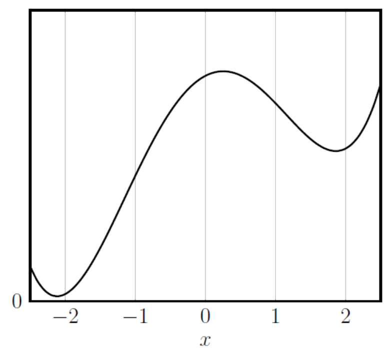
$y = f'(x)$



$y = f''(x)$



7. The graph of $y = f'(x)$ is shown. Where is $f(x)$ greatest? Least? Where is $f'(x)$ greatest? Least? Where is $f''(x)$ greatest? Least? Where is $f(x)$ concave up? Concave down?



Solution: $f'(x)$ is positive for the entire interval, so $f(x)$ is greatest at 2.5 and least at -2.5. Looking at the peaks and valleys of the graph of $f'(x)$ given, $f'(x)$ is greatest at $x = 0.2$ and least at $x = -2.1$. Looking at the slope of $f'(x)$, we estimate $f''(x)$ is greatest at $x = -1$ and $x = 2.5$, and $f''(x)$ is least at $x = -2.5$. $f(x)$ is concave up where $f'(x)$ is increasing, from about -2.1 to 0.2 and about 1.9 to 2.5. $f(x)$ is concave down where $f'(x)$ is decreasing, from about -2.5 to -2.1 and from about 0.2 to 1.9.

8. For what values of x is the graph of $y = x^5 - 5x$ both increasing and concave up?

Solution: $x > 1$

9. Where does the tangent line to $y = 2^x$ through $(0, 1)$ intersect the x-axis?

Solution: $\left(-\frac{1}{\ln 2}, 0\right)$

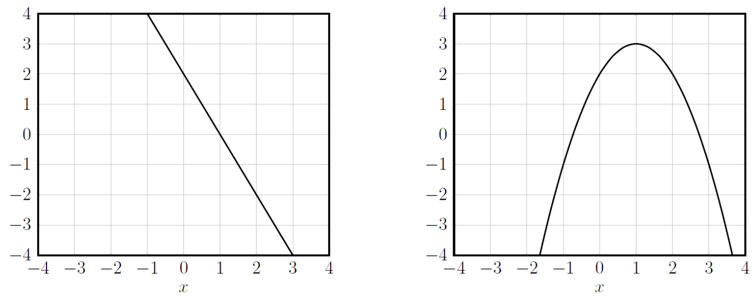
10. If $g(x) = e^x f(x)$, find and simplify $g''(x)$.

Solution: $g''(x) = e^x f(x) + 2e^x f'(x) + e^x f''(x)$

11. If $f(x) = 4x^3 + 6x^2 - 23x + 7$, find the intervals on which $f'(x) \geq 1$.

Solution: $[1, \infty)$ and $(-\infty, -2]$.

12. Let $f(x)$ and $g(x)$ be the functions graphed below:



Let $h(x) = f(x)g(x)$, let $j(x) = x^2 f(x)$, let $k(x) = f(x^2)$, let $p(x) = \frac{f(x)}{g(x)}$, let $q(x) = f(g(x))$, and $r(x) = g(g(x))$.

(a) Estimate $h'(1), h'(0), p'(0), q'(0), r'(1)$, and $r'(2)$.

Solution: $h'(1) = -6, h'(0) = 0, p'(0) = -2, q'(0) = -4, r'(1) = 0, r'(2) = 4$

(b) Estimate $j'(-1)$ and $k'(-1)$.

Solution: $j'(-1) = -10$ and $k'(-1) = 4$

(c) Estimate all values of x for which $y = r(x)$ has a horizontal tangent line.

Solution: $x = 1, x = -0.4, x = 2.4$

13. Using the information in the table, find:

x	1	2	3	4
$f(x)$	3	2	1	4
$f'(x)$	1	4	2	3
$g(x)$	2	1	4	3
$g'(x)$	4	2	3	1

(a) $h(4)$ if $h(x) = f(g(x))$

Solution: $h(4) = 1$

(b) $h'(4)$ if $h(x) = f(g(x))$

Solution: $h'(4) = 2$

(c) $h(4)$ if $h(x) = g(f(x))$

Solution: $h(4) = 3$

(d) $h'(4)$ if $h(x) = g(f(x))$

Solution: $h'(4) = 3$

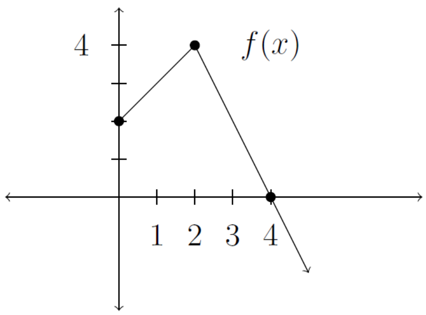
(e) $h'(4)$ if $h(x) = \frac{g(x)}{f(x)}$

Solution: $h'(4) = -\frac{5}{16}$

(f) $h'(4)$ if $h(x) = f(x)g(x)$

Solution: $h'(4) = 13$

14. A graph of $f(x)$ is shown below. It is piecewise linear. The table below gives values of $g(x)$ and $g'(x)$.



x	0	1	2	3	4
$g(x)$	2	5	9	11	8
$g'(x)$	3	4	2	-3	-4

(a) Given $h(x) = f(x)g(x)$, find $h'(1)$.

Solution: $h'(1) = 17$

(b) Given $k(x) = \frac{f(x)}{g(x)}$, find $k'(3)$.

Solution: $k'(3) = -\frac{16}{121}$

(c) Given $l(x) = \frac{g(x)}{\sqrt{x}}$, find $l'(4)$.

Solution: $l'(4) = -\frac{5}{2}$

(d) Given $m(x) = g(f(x))$, find $m'(3)$.

Solution: $m'(3) = -4$

15. On what interval(s) is the function

$$f(x) = \frac{(5x+2)^3}{(2x+3)^3}$$

increasing?

Solution: $\left(-\infty, -\dfrac{3}{2}\right) \cup \left(-\dfrac{3}{2}, \infty\right)$

16. If $g(2) = 3$ and $g'(2) = -4$, find $f'(2)$ for the following:

(a) $f(x) = x^2 - 4g(x)$

Solution: $f'(2) = 20$

(b) $f(x) = \frac{x}{g(x)}$

Solution: $f'(2) = \frac{11}{9}$

(c) $f(x) = x^2 g(x)$

Solution: $f'(2) = -4$

(d) $f(x) = g(x)^2$

Solution: $f'(2) = -24$

(e) $f(x) = x \sin(g(x))$

Solution: $f'(2) = \sin(3) - 8\cos(3)$

(f) $f(x) = x^2 e^{g(x)}$

Solution: $f'(2) = -12e^3$

17. On what interval(s) is the function

$$f(x) = (x+3)e^{2x}$$

decreasing? On what intervals is it concave down?

Solution: Decreasing on $\left(-\infty, -\dfrac{7}{2}\right)$. Concave down on $(-\infty, -4)$.

18. Let $f(x) = xe^x$.

(a) Find the interval(s) where f is increasing and the interval(s) where f is decreasing.

Solution: $f'(x) = xe^x + xe^x = e^x(1+x)$. Recall that $e^x > 0$ for all x. Then we know f is increasing when $f'(x) > 0$ so $1 + x >$ so $x > -1$ (i.e. on the interval $(-1, \infty)$). f is decreasing when $f'(x) < 0$ so $1 + x < 0$ so $x < -1$ (i.e. on the interval $(-\infty, -1)$.)

(b) Find the interval(s) where f is concave up and the interval(s) where f is concave down.

Solution: $f''(x) = e^x(x+2)$. Recall that $e^x > 0$ for all x. Then we know f is concave up when $f''(x) > 0$ so $x + 2 >$ so $x > -2$ (i.e. on the interval $(-2, \infty)$). f is concave down when $f''(x) < 0$ so $x + 2 < 0$ so $x < -2$ (i.e. on the interval $(-\infty, -2)$.)

19. Find the slope of the line tangent to $y = \cos(3\theta)$ at $\theta = -\dfrac{\pi}{18}$

Solution: $y' = -3\sin(3\theta)$. When we evaluate this at $\theta = -\frac{\pi}{18}$, we see the derivative, y', (i.e. slope of the line tangent) at $\theta = -\frac{\pi}{18}$ is equal to $\frac{3}{2}$.

20. Differentiate the following functions.

(a) $a(x) = x^4 + 2x^3 + 6x$

Solution: $a'(x) = 4x^3 + 6x^2 + 6$

(b) $b(x) = \sqrt{x} + \dfrac{1}{3x^2}$

Solution: $b'(x) = \dfrac{1}{2\sqrt{x}} - \dfrac{2}{3x^3}$

(c) $c(x) = (x^3 - x)e^x$

Solution: $c'(x) = (3x^2 - 1)e^x + (x^3 - x)e^x$

(d) $d(t) = \dfrac{t+2}{t+1}$

Solution: $d'(t) = \dfrac{(t+1)\cdot 1 - (t+2)\cdot 1}{(t+1)^2}$

(e) $k(p) = \sec\left(e^{(p^2+2)}\right)$

Solution: $k'(p) = \sec\left(e^{(p^2+2)}\right)\tan\left(e^{(p^2+2)}\right)\cdot e^{(p^2+2)}\cdot 2p$

(f) $d(t) = 2e^t + t^{2/5} - 2t^5 + t^{-3} - \pi^2$

Solution: $d'(t) = 2e^t + \frac{2}{5}t^{-3/5} - 10t^4 - 3t^{-4}$

(g) $h(x) = \dfrac{-4x^2 + 3x - 1}{4x^3 + 10}$

Solution: $h'(x) = \dfrac{(4x^3+10)(-8x+3) - (-4x^2+3x-1)(12x^3)}{(4x^3+10)^2}$

(h) $f(\theta) = \sec(\theta^2 + \cos(\theta))$

Solution: $f'(\theta) = \sec(\theta^2 + \cos(\theta))\tan(\theta^2 + \cos(\theta))(2\theta - \sin(\theta))$

(i) $n(w) = 4^w(w^2 + 11)$

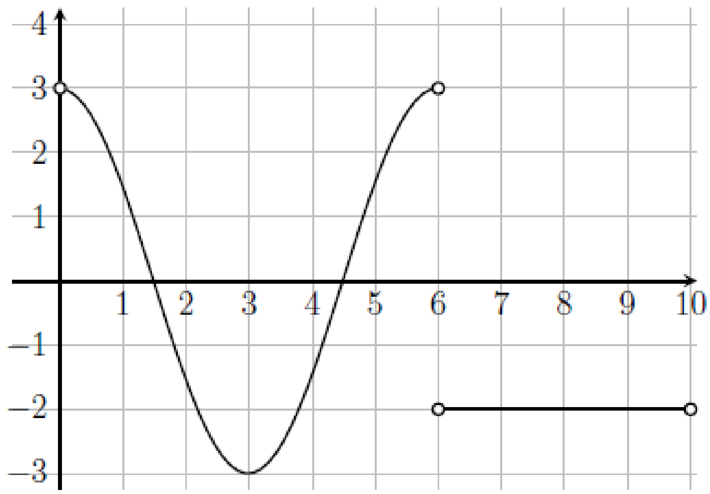
Solution: $n'(w) = \ln(4)4^w(w^2 + 11) + 4^w(2w)$

21. Find the point on the graph of $f(x) = 3x^2 - 4x + 5$ such that the tangent line at that point is parallel to the line $y = 2x + 150$.

Solution: (1,4)

22. The graph below is the **derivative** of some function, f.

Graph of $f'(x)$



(a) On what intervals is f increasing?

Solution: $(0, 1.5) \cup (4.5, 6)$

(b) At what values of x does $f(x)$ have a local minimum?

Solution: $x = 4.5$

(c) On what intervals is $f(x)$ concave up?

Solution: (3,6)

(d) At what values of x does $f(x)$ have an inflection point?

Solution: $x = 3$

(e) Is it possible for the function $f(x)$ to be continuous at $x = 6$? Explain.

Solution: Yes. At $x = 6$ the function $f(x)$ is not differentiable, but this does not imply it is not continuous. For example, there could be a corner or a cusp at $x = 6$.

23. Let $f(x) = x^3 + \dfrac{9}{2}x^2 - 12x + 13$.

    (a) Find $f'(x)$ and $f''(x)$.

    Solution: $f'(x) = 3x^2 + 9x - 12$ and $f''(x) = 6x + 9$

    (b) On what interval(s) is f decreasing?

    Solution: (-4,1)

    (c) On what interval(s) is f concave downward?

    Solution: $(-\infty, -3/2)$

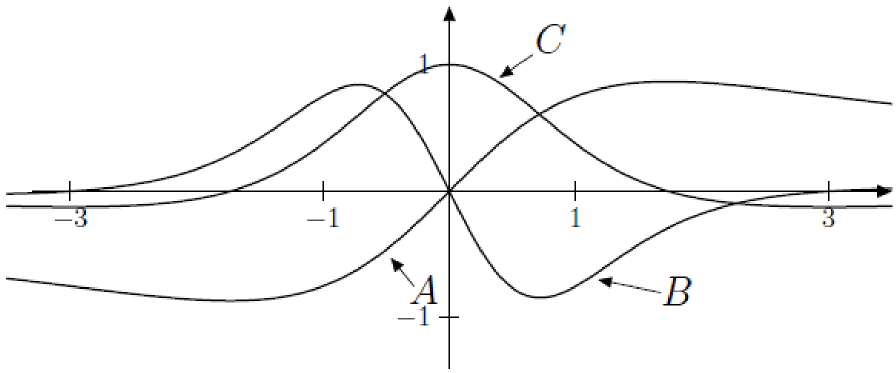
24. Consider the curve described by the points satisfying the equation

$$x^3 + y^3 = 2x^3y + 5$$

Find the equation of the tangent line at the point (1,2).

Solution: $(y-2) = \frac{9}{10}(x-1)$

25. The graph of a function $f(x)$ and its first and second derivatives are shown below.



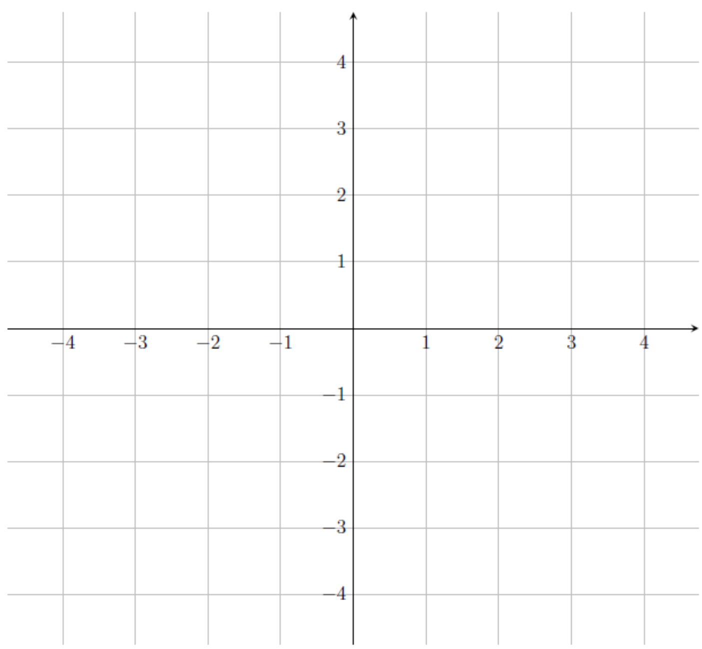
Explain which graph is $f(x)$, $f'(x)$, and $f''(x)$.

Solution: Graph A represents the function $f(x)$. Graph C represents the first derivative function $f'(x)$, and Graph B represents the second derivative function $f''(x)$.

26. Draw the graph of a function $f(x)$ that satisfies the following conditions:

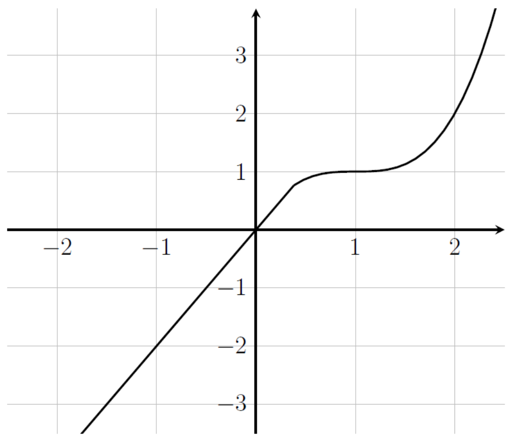
- $f(x) < 0$ when $x < -3$
- $f(x) > 0$ on the intervals $(-3, 0)$ and $(0, \infty)$
- $f(-3) = f(0) = 0$
- $f'(x) > 0$ when $x < -2$ and $x > 0$
- $f'(x) < 0$ on the interval $(-2, 0)$
- $f'(-2) = f'(0) = 0$
- $f''(x) > 0$ when $-1 < x < 2$
- $f''(x) < 0$ on the intervals $(-\infty, -1)$ and $(2, \infty)$
- $f''(-1) = f''(2) = 0$

Graph of $f(x)$



27. Find the derivatives of the following functions at the given point using the information given below:

Graph of $f(x)$



x	-3	-2	-1	0	1	2	3	4
$g(x)$	2	-4	-2.5	1	3	5	14	5
$g'(x)$	-3	-2	2	4	2	4	3	-6

(a) If $h(x) = g(g(x))$, what is $h'(-3)$?

Solution: $h'(3) = -12$

(b) If $q(x) = f(x)g(x)$, what is $q'(1)$?

Solution: $q'(1) = 2$

(c) If $m(x) = g(\sqrt{x})$, what is $m'(4)$?

Solution: $m'(4) = 1$

28. Hercules is attempting to scale a Roman wall, so he leans a 13 meter ladder against the vertical wall. While he is climbing the ladder, the goddess Juno kicks the bottom of the ladder away from the wall causing the top of the ladder to start sliding down the wall. When the top of the ladder is 5 meters above the ground, the top of the ladder is sliding down the wall at a rate of 2 meters per second. At this same point in time, what is the speed of the bottom of the ladder as it moves away from the wall? Be sure to include units in your answer.

Solution: Let s be the measurement along the hypotenuse of the right triangle, which is 13 m. We are given $dy/dt = -2$ m/s, the rate the ladder slides down the wall to be 2 m/s. We also know that at time t the top of the ladder is 5 m above the ground, and the bottom of the ladder is 12 m from the wall (by the Pythagorean Theorem). Our goal is to find dx/dt. Recall that $x^2 + y^2 = s^2$. Differentiating with respect to time we get

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2s\frac{ds}{st}.$$

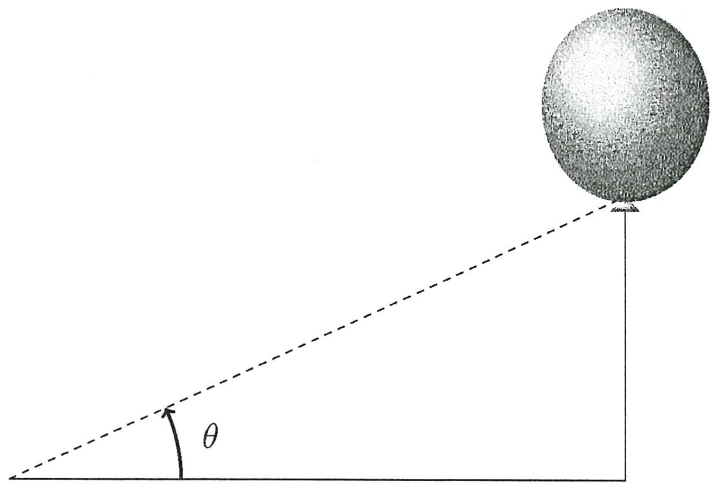
Since our hypotenuse remains constant, $\frac{ds}{dt} = 0$, so we have

$$\frac{dx}{dt} = -\frac{y}{x}\frac{dy}{dt}.$$

When $y = 5$, $x = 12$, and $dy/dt = -2$, we get

$$\frac{dx}{dt} = \frac{5}{6} \text{ m/s}.$$

29. An observer stands 200 m from the launch site of a hot-air balloon. The balloon rises vertically at a constant rate of 4 m/s. How fast is the angle of elevation of the balloon increasing 50 s after launch?



Solution: We have the right triangle with the angle θ at the vertex of our observer. The observer stands 200 m from the launch site, and we can denote the height of the balloon at time t to be y. Then we can write

$$\tan(\theta) = \frac{y}{200}$$

We know the rate the balloon is rising is $\frac{dy}{dt} = 4$ m/s. The rate we want to find is $\frac{d\theta}{dt}$. Using implicit differentiation on our the equation $\tan(\theta) = \dfrac{y}{200}$ we get

$$\sec^2(\theta)\frac{d\theta}{dt} = \frac{1}{200}\frac{dy}{dt}$$
$$\frac{d\theta}{dt} = \frac{1}{200\sec^2(\theta)}\frac{dy}{dt}$$

Since at $t = 50$ sec, we know the height of the balloon will be 200 m, we can see that we have a right isosceles triangle with two acute angle measurements of $\pi/4$. So our $\theta = \pi/4$. Then $\sec^2(\pi/4) = 2$. Solving for $\frac{d\theta}{dt}$ we have

$$\frac{d\theta}{dt} = \frac{1}{200(2)} \cdot 4 = \frac{1}{100}\frac{\text{radians}}{\text{second}}$$

Remember that radians are unitless. So getting units of $\frac{1}{\text{sec}}$ is the same as $\frac{\text{rad}}{\text{sec}}$.

30. A street light is mounted at the top of a 15 ft tall pole. A boy 5 ft tall walks away from the pole at a speed of 3 ft/sec along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole?

Solution: To solve this problem, we will need to recall the idea of similar triangles from geometry. This idea will come into play since we form two right triangles using the light pole and the boy as vertical sides. Let x be the distance between the pole and the boy, y bet the length of the boy's shadow, and $x+y$ be the total distance along the base of the right triangle formed by the light of the pole. Note both x and y depend on time as the boy moves away from the pole. We can then setup the ratio $\frac{x+y}{15} = \frac{y}{5}$ using the idea of similar triangles. This can be simplified to $15y = 5x + 5y$ or $10y = 5x$. Now using implicit differentiation we get $10\frac{dy}{dt} = 5\frac{dx}{dt}$. Note that $\frac{dy}{dt}$ represents the rate the tip of the shadow moves away from the boy. Since we know the boy is moving away from the pole at 3 ft/sec, we know $\frac{dx}{dt} = 3$. Now solving for $\frac{dy}{dt}$ we get the shadow is moving 3/2 ft/sec from the boy. To find the speed the tip of the shadow is moving from the light pole, we add the boy's speed and the shadow's speed together, which gives us the tip of the shadow is moving 4.5 ft/sec away from the pole when the boy is 40 ft from the pole.

31. Given the curve $e^y = 2x^3y^2 + e^2$, find $\frac{dy}{dx}$ in terms of x and y.

Solution: $y' = \dfrac{6x^2y^2}{e^y - 4x^3y}$

32. If $f(x) = x^2 + 1$ and $g(x) = 5 - x$, find:

(a) $h'(1)$ if $h(x) = f(x) \cdot g(x)$

Solution: $h'(1) = 6$

(b) $j'(2)$ if $j(x) = \frac{f(x)}{g(x)}$

Solution: $j'(2) = \frac{17}{9}$

(c) $k'(3)$ if $k(x) = f(g(x))$

Solution: $k'(3) = -4$

33. Find the equations for the lines tangent to the graph of $xy + y^2 = 4$ when $x = 3$.

Solution: The equations are

$$y - 1 = -\tfrac{1}{5}(x-3) \text{ and } y + 4 = -\tfrac{4}{5}(x-3)$$

34. Find $\frac{dy}{dx}$ if $x^3 + y^3 - 4x^2y = 0$.

Solution: $\dfrac{dy}{dx} = \dfrac{8xy - 3x^2}{3y^2 - 4x^2}$

35. Consider the curve $x^2 + 2xy + 5y^2 = 4$. At what point(s) is the tangent line to this curve horizontal? At what point(s) is the tangent line to this curve vertical? At what points is the slope of the tangent line equal to 2?

Solution: The derivative is $\dfrac{dy}{dx} = -\dfrac{x+y}{x+5y}$. The tangent line is horizontal when $y = -x$, which happens at the points (1,-1) and (-1,1). The tangent line is vertical when $x = -5y$, which happens at the points $(-\sqrt{5}, \frac{\sqrt{5}}{5})$ and $(\sqrt{5}, -\frac{\sqrt{5}}{5})$. The slope of the tangent line equals 2 when $x = -\frac{11}{3}y$, which happens at the points $(-\frac{11}{5}.\frac{3}{5})$ and $(\frac{11}{5}.-\frac{3}{5})$

36. Consider the function $y + \sin(y) + x^2 = 9$. Find the slope of the curve at $(3, 0)$.

Solution: The slope is -3.

37. A radio navigation system used by aircraft gives a cockpit readout of the distance, s, in miles, between a fixed ground station and the aircraft. The system also gives a readout of the instantaneous rate of change, $\frac{ds}{dt}$, of this distance in miles/hour. An aircraft on a straight flight path at a constant altitude of 10,560 feet (2 miles) has passed directly over the ground station and is now flying away from it. What is the speed of this aircraft along its constant altitude flight path when the cockpit readouts are s=4.6 miles and $\frac{ds}{dt}$=210 miles/hour?

Solution: s is measured along the hypotenuse of the right triangle; we are given ds/dt and we want to find dx/dt, where $x^2 + 2^2 = s^2$. Differentiating with respect to time we get

$$2x\frac{dx}{dt} = 2s\frac{ds}{st}$$

so that

$$\frac{dx}{dt} = \frac{s}{x}\frac{ds}{dt} = \frac{s}{\sqrt{s^2-4}}\frac{ds}{dt}.$$

When $s = 4.6$ and $ds/dt = 210$, we get

$$\frac{dx}{dt} = 233 \text{ miles per hour.}$$

38. A gas station stands at the intersection of a north-south road and an east-west road. A police car is traveling toward the gas station. The speed of the police car is 100 mph at the moment it is 3 miles from the gas station. At the same time, the truck is 4 miles from the gas station going 80 mph. At this moment, is the distance between the car and truck increasing or decreasing? How fast? (Distance is measured along a straight line joining the car and truck.)

Solution: The distance is increasing at 4 miles per hour.

39. Is

$$f(x) = \begin{cases} 2x+1 & ,\text{when } x \geq 3 \\ x^2+2 & ,\text{when } x < 3 \end{cases}$$

differentiable at $x = 3$?

Solution: $f(x)$ is not continuous at $x = 3$, which implies that it is not differentiable at at $x = 3$.

40. Is

$$f(x) = \begin{cases} 2x+1 & ,\text{when } x < 0 \\ (x+1)^2 & ,\text{when } x \geq 0 \end{cases}$$

differentiable at $x = 0$?

Solution: Check to show $f(x)$ is continuous at $x = 0$. We then verify the limits of the difference quotients (definition of derivative) from left and right. These limits are equal, so $f(x)$ is differentiable at $x = 0$. The graph is smooth, so f is differentiable at $x = 0$.

41. Is

$$f(x) = \begin{cases} x+1 & ,\text{when } x \leq 0 \\ 1-x^2 & ,\text{when } x > 0 \end{cases}$$

differentiable at $x = 0$?

Solution: Check to show $f(x)$ is continuous at $x = 0$. Then check the limits of the difference quotients (definition of derivative) from left and right. These limits are not equal, so $f(x)$ is not differentiable at $x = 0$. The graph is has a cusp, so f is not differentiable at $x = 0$.

42. Is $f(x) = x^{\frac{1}{3}}$ differentiable at $x = 0$?

Solution: Check to show $f(x)$ is continuous at $x = 0$. Then check the limits of the difference quotients (definition of derivative) from left and right. These limits are equal, but notice there limits equal ∞. The graph is has a vertical tangent line, so f is not differentiable at $x = 0$.

Derivative Practice

Find the derivative of each function.

1. $f(x) = \dfrac{x^2+1}{5}$

   Solution: $f'(x) = \dfrac{2}{5}x$

2. $f(x) = \pi^3$

   Solution: $f'(x) = 0$

3. $r(t) = \sqrt{x} + \frac{1}{3q}$

   Solution: $r'(t) = \dfrac{1}{2\sqrt{x}} - \dfrac{1}{3x^2}$

4. $c(q) = \dfrac{1+q^2+q^3+q^4+q^5+q^6}{q^3}$

   Solution: $c'(q) = -\dfrac{3}{x^4} - \dfrac{1}{x^2} + 1 + 2x + 3x^2$

5. $k(x) = \dfrac{1}{x^2}$

   Solution: $k'(x) = -\dfrac{2}{x^3}$

6. $g(p) = (p+1)(2p-1)$

   Solution: $g'(p) = 4p + 1$

7. $f(x) = xe^x$

   Solution: $f'(x) = (x+1)e^x$

8. $y(x) = \sin(x)\cos(x)$

   Solution: $y'(x) = \cos^2(x) - \sin^2(x)$

9. $f(x) = 13(2^{x-3}) + x^{2/3} - x^{-3/2}$

   Solution: $f'(x) = 13(2^{x-3}\ln 2) + \frac{2}{3}x^{-1/3} + \frac{3}{2}x^{-5/2}$

10. $m(x) = kx^n$

    Solution: $m'(x) = knx^{n-1}$

11. $f(x) = \dfrac{2x-1}{x+3}$

    Solution: $f'(x) = \dfrac{7}{(x+3)^2}$

12. $c(t) = t^2\cos(t) + 4\sin(t)$

    Solution: $c'(t) = 2t\cos(t) - t^2\sin(t) + 4\cos(t)$

13. $s(t) = \dfrac{\sin(t)\cos(t)}{1 + t\tan(t)}$

    Solution: $s'(t) = \dfrac{(\cos^2(t) - \sin^2(t))(1 + t\tan(t)) - (\sin(t)\cos(t))(\tan(t) + t\sec^2(t))}{(1 + t\tan(t))^2}$

14. $h(x) = \cos^2(x) + \sin^2(x)$

    Solution: $h'(x) = 0$

15. $P(t) = 2\sin^2(t)$

    Solution: $P'(t) = 4(\sin(t))(\cos(t))$

16. $f(x) = \dfrac{\sec(x)}{1 + \tan(x)}$

    Solution: $f'(x) = \dfrac{(\tan(x)\sec(x))(1 + \tan(x)) - (\sec(x))(\sec^2(x))}{(1 + \tan(x))^2}$

17. $s(t) = \dfrac{t^2 + 1}{t + 1}$

    Solution: $s'(t) = \dfrac{t^2 + 2t - 1}{(t + 1)^2}$

18. $R(q) = (2q^7 - q^2) \cdot \dfrac{q - 1}{q + 1}$

    Solution: $R'(q) = (14q^6 - 2q) \cdot \dfrac{q - 1}{q + 1} + (2q^7 - q^2) \cdot \dfrac{(2)}{(q + 1)^2}$

19. $P(t) = 432e^{-0.043t}$

    Solution: $P'(t) = -18.576e^{-0.043t}$

20. $f(x) = 2e^{\cos(3x^2)}$

    Solution: $f'(x) = -12x\sin(3x^2)e^{\cos(3x^2)}$

21. $y = \dfrac{e^{10x^2}\sin(20x)}{\sqrt{x^2+3\cos(x)-1}}$

Solution:

$$y' = \frac{(20xe^{10x^2}\sin(20x) + 20e^{10x^2}\cos(x))(\sqrt{x^2+3\cos(x)-1})}{x^2+3\cos(x)-1}$$
$$- \frac{e^{10x^2}\sin(20x)(\frac{1}{2})(x^2+3\cos(x)-1)^{-1/2}(2x-3\sin(x))}{x^2+3\cos(x)-1}$$